From Almost-Commutative Geometry to Almost-Associative Geometry

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Overview

- Non-Associative Geometry:
  - What?
  - Why?
  - How?

Non-Associative Geometry: Why?

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- Mathematical Reason:

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  - Groups: Non-Abelian
  - Algebras: Non-Associative

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$$
\begin{aligned}
SO(10) &\to SU(4) \times SU(2) \times SU(2) \\
&\to SU(3) \times SU(2) \times U(1)
\end{aligned}
$$

$$
\begin{aligned}
16 &\to (4,2,1) + (\bar{4},1,2) \\
&\to (3,2,1/6) + (\bar{3},1,-2/3) + (\bar{3},1,1/3) \\
&\qquad +(1,2,-1/2) + (1,1,0) + (1,1,+1)
\end{aligned}
$$

Non-Associative Algebras

$$[a, b] \equiv ab - ba$$ ("commutator")

$$[a, b, c] \equiv (ab)c - a(bc)$$ ("associator")

Jordan	$h_n(\mathbb{R})$	$h_n(\mathbb{C})$	$h_n(\mathbb{H})$	$h_3(\mathbb{O})$	$\delta_{a,b} = [L_a, R_b]$
Normed Division	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$	$\delta_{a,b} = [L_a, R_b] + [L_a, L_b] + [R_a, R_b]$
Lie	$a_n^0(\mathbb{R})$	$a_n^0(\mathbb{C})$	$a_n(\mathbb{H})$	$\{\dots\}$	$\delta_a = L_a - R_a$

$$\alpha = e^{\delta}$$

$\alpha(ab) = \alpha(a)\alpha(b)$	$\alpha(a^*) = \alpha(a)^*$
$\delta(ab) = \delta(a)b + a\delta(b)$	$\delta(a^*) = \delta(a)^*$

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- How to "represent" a non-associative algebra?
- How to "fluctuate" D?
- Simplest non-trivial geometries.
- Correspond to *ordinary* Higgs-Yang-Mills theories (coupled to gravity)
- What next?