some physical implications of almost commutative manifolds

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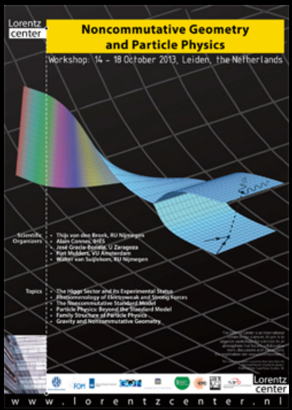
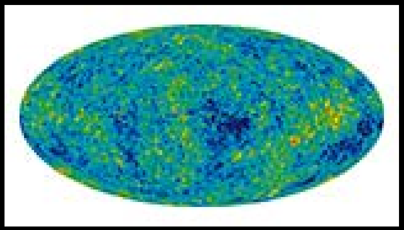


Lorentz center

cosmology

EU models tested with

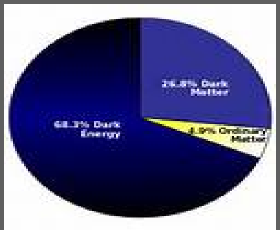
- astrophysical data (CMB)



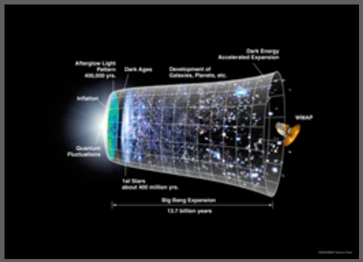
- high energy experiments (LHC)

despite the golden era of cosmology, a number of questions:

- origin of DE / DM



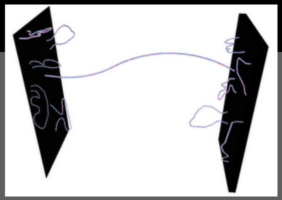
- search for natural and well-motivated inflationary model (alternatives...)



. . .

are still awaiting for a definite answer main approaches:

- string theory



- LQC, SF, WdW, CDT, CS,...

- noncommutative spectral geometry

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\ d^4x\ ,$$

$$Tr(f(D_A/\Lambda))$$

f : cut-off function ⇛ its taylor expansion at zero vanishes

⇛ the asymptotic expansion of the trace reduces to:

$$\boxed{\mathrm{Tr}\left(f\left(\frac{D_A}{\Lambda}\right)\right) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4}$$

f plays a rôle through its momenta f_0, f_2, f_4

real parameters related to the coupling constants at unification, the gravitational constant, and the cosmological constant

- full SM lagrangian
- majorana mass terms for right-handed neutrinos
- gravitational terms coupled to matter

> EH action with a cosmological term
> topological term
> conformal gravity term with the weyl curvature tensor
> conformal coupling of higgs to gravity

the coefficients of the gravitational terms depend upon the yukawa parameters of the particle physics content

$$S^{\rm bosonic} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^* R^* + \frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} + \frac{1}{4} F^{\alpha}_{\mu\nu} F^{\alpha\mu\nu}$$
$$+ \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x ,$$

bare action a la wislon

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}},$$
$$\alpha_0 = -\frac{3 f_0}{10\pi^2},$$
$$\gamma_0 = \frac{1}{\pi^2}\left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4} \mathfrak{d}\right),$$
$$\tau_0 = \frac{11 f_0}{60\pi^2},$$
$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}},$$
$$\xi_0 = \frac{1}{12},$$
$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2},$$

$$\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$$

$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ describe possible choices of $D_{\mathcal{F}}$

yukawa parameters and majorana terms for $\nu_{\rm R}$

gravitational & coupling between Higgs field and Ricci curvature

⇒ equations of motion

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\beta^2}\delta_{\rm cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{\rm cc}T^{\mu\nu}_{\rm matter}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{\rm cc} \equiv [1 - 2\kappa_0^2\xi_0\mathbf{H}^2]^{-1}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

nelson, sakellariadou, PRD 81 (2010) 085038

gravitational & coupling between Higgs field and Ricci curvature

⇒ equations of motion

neglect nonminimal coupling between geometry and higgs

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{\beta^2}\delta_{cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2 \delta_{cc} T^{\mu\nu}_{\mathrm{matter}}$$

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FLRW

weyl tensor vanishes ⇒

NCSG corrections to einstein equations vanish

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gravitational & coupling between Higgs field and Ricci curvature

⟹ equations of motion

neglect nonminimal coupling between geometry and higgs

⟹ corrections to einstein's eqs. will be apparent at leading order, only in anisotropic models

bianchi model: NCSG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic spacetimes

nelson, sakellariadou, PRD 81 (2010) 085038

$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

$$A_i(t) = \ln a_i(t)$$

$$\begin{aligned}
&\kappa_0^2 T_{00} = \\
&-\dot{A}_3\left(\dot{A}_1 + \dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right) \\
&+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\Big[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 \\
&-\dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\Big] \\
&-\frac{4\alpha_0\kappa_0^2}{3}\sum_i \Big\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i \\
&+\dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i - \dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right) \\
&+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\Big[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right) \\
&+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\Big] \\
&+\Big[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\Big] \\
&\times\Big[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\Big]\Big\}
\end{aligned}$$

nelson, sakellariadou, PRD 81 (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2/6} \right] T^{\mu\nu}_{\mathrm{matter}}$$

⇨ effective gravitational constant

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}| g_{\alpha\beta} - \mu_0 |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4$$

$$\Rightarrow \quad -\mu_0 |\mathbf{H}|^2 \to -\left(\mu_0 + \frac{R}{12}\right) |\mathbf{H}|^2$$

⇨ increases the higgs mass

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remarks

- redefine higgs: $\tilde{\phi} = -\ln\left(|\mathbf{H}|/(2\sqrt{3})\right)$

⇨ rewrite higgs lagrangian in terms of 4dim dilatonic gravity

$$\mathcal{L}_{\tilde{\phi}} = e^{-2\tilde{\phi}}\left[-R + 6D^{\alpha}\tilde{\phi}D^{\beta}\tilde{\phi}g_{\alpha\beta} - 12\left(\mu_0 - 12\lambda_0 e^{-2\tilde{\phi}}\right)\right]$$

link with compactified string models

- chameleon models
  scalar field with nonminimal coupling to standard matter

  NCSG
  scalar field (higgs) with nonzero coupling to bckg geometry
  mass & dynamics of higgs dependent on local matter content

link with chameleon cosmology

bosonic

$$S = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^* R^* + \frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\alpha\mu\nu} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \right) \sqrt{g}\, d^4x,$$

bare action a la wislon

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}},$$

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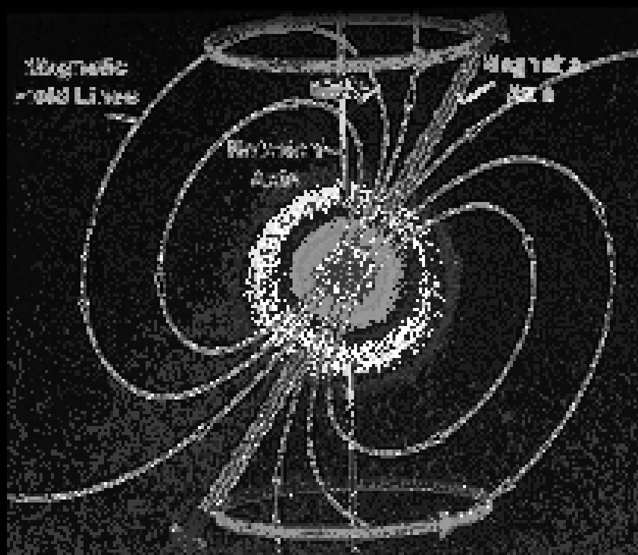
yukawa parameters and majorana terms for ν_R

gravitational waves in NCSG



nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

lambiase, sakellariadou, stabile, arXiv:1302.2336 linear perturbations around minkowski background in synchronous gauge:

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, (\delta_{ij} + h_{ij}(x))\right]\right)$$

$a(t) = 1$ $\nabla_i h^{ij} = 0$

$$\boxed{\left(\Box - \beta^2\right)\Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\mathrm{matter}}}$$

with conservation eqs: $\frac{\partial}{\partial x^\mu} T^\mu{}_\nu = 0$

$$\beta^2 = -\frac{1}{32\pi G \alpha_0} \qquad \alpha_0 = \frac{-3f_0}{10\pi^2}$$

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021 energy lost to gravitational radiation by orbiting binaries:

$$\boxed{-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}}$$

strong deviations from GR at frequency scale

$$2\omega_c \equiv \quad \beta c \sim (f_0 G)^{-1/2} c$$

set by the moments of the test function f

scale at which NCSG effects become dominant

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021 restrict β by requiring that the magnitude of deviations from GR must by less than the uncertainty

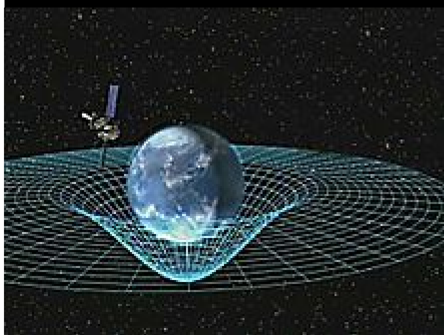
Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
PSR J0737-3039	~ 500	2.454	0.088	0.2
PSR J1012-530[illegible]	[illegible]	[illegible]	[illegible]10^{-6}	10
PSR J1141-654[illegible]	[illegible]	[illegible]	[illegible].17	6
PSR B1916+16	~ 6400	7.752	0.617	0.1
PSR B1534+12	~ 1100	10.1	?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

$$\beta > 7.55 \times 10^{-13}\mathrm{m}^{-1}$$

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

accuracy to which the rate of change of orbital period agrees with predictions of GR gravity probe B

the satellite contains a set of gyroscopes in low circular polar orbit with altitude h=650 km



geodesic precession in the orbital plane
lense-thirring (frame dragging) precession in the plane of earth equator

GR

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

milliarcsec/yr

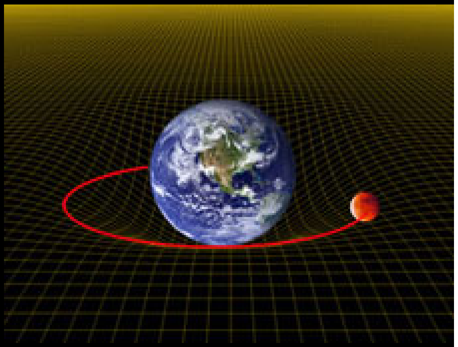
e.o.m. for gyro spin 3 vector S:

$$\frac{d\mathbf{S}}{dt} = \frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} + \frac{d\mathbf{S}}{dt}\Big|_{\mathrm{LT}}$$

metric: $$ds^2 = -(1+2\Phi)dt^2 + 2\mathbf{A}\cdot d\mathbf{x}dt + (1+2\Psi)d\mathbf{x}^2$$

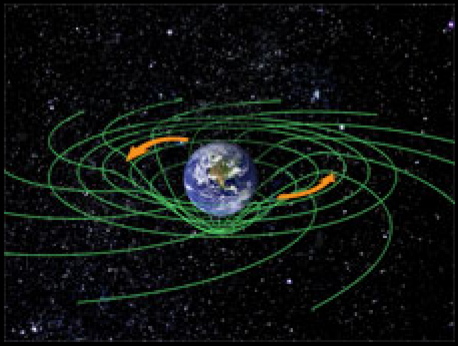
instantaneous geodesic precession

$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} = \mathbf{\Omega}_{\mathrm{G}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{G}} = \frac{1}{2}[\nabla(\Phi - 2\Psi)] \wedge \mathbf{v}$$



instantaneous lense-thirring precession

$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{LT}} = \mathbf{\Omega}_{\mathrm{LT}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{LT}} = \frac{1}{2}\nabla \wedge \mathbf{A}$$



instantaneous geodesic precession

$$\Omega_{\rm geodesic} = \Omega_{\rm geodesic(GR)} + \Omega_{\rm geodesic(NCG)}$$

$$\boxed{\Omega_{\rm geodesic(GR)} = 6606 \ \ {\rm mas/y}}$$

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

require $$|\Omega_{\rm geodesic(NCG)}| \leq \delta\Omega_{\rm geodesic}$$

with $$\delta\Omega_{\rm geodesic} = 18 \ \ {\rm mas/y}$$

$$\Longrightarrow \quad \boxed{\beta \gtrsim 1.1 \times 10^{-6} {\rm m}^{-1}}$$

lambiase, sakellariadou, stabile, arXiv:1302.2336

inflation through the nonminimal coupling between the geometry and the higgs field

proposal: the higgs field, could play the rôle of the inflaton

but

GR: to get the amplitude of density perturbations, the higgs mass would have to be 11 orders of magnitude **higher**

re-examine the validity of this statement within NCSG

nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

$$S^{\mathrm{L}}_{\mathrm{GH}} = \int \Big[\frac{1 - 2\kappa_0^2 \xi_0 H^2}{2\kappa_0^2} R - \frac{1}{2}(\nabla H)^2 - V(H)\Big] \sqrt{-g}\; d^4x$$

$$V(H) = \lambda_0 H^4 - \mu_0^2 H^2$$

subject to radiative corrections as a function of energy

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}$$

$$\xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

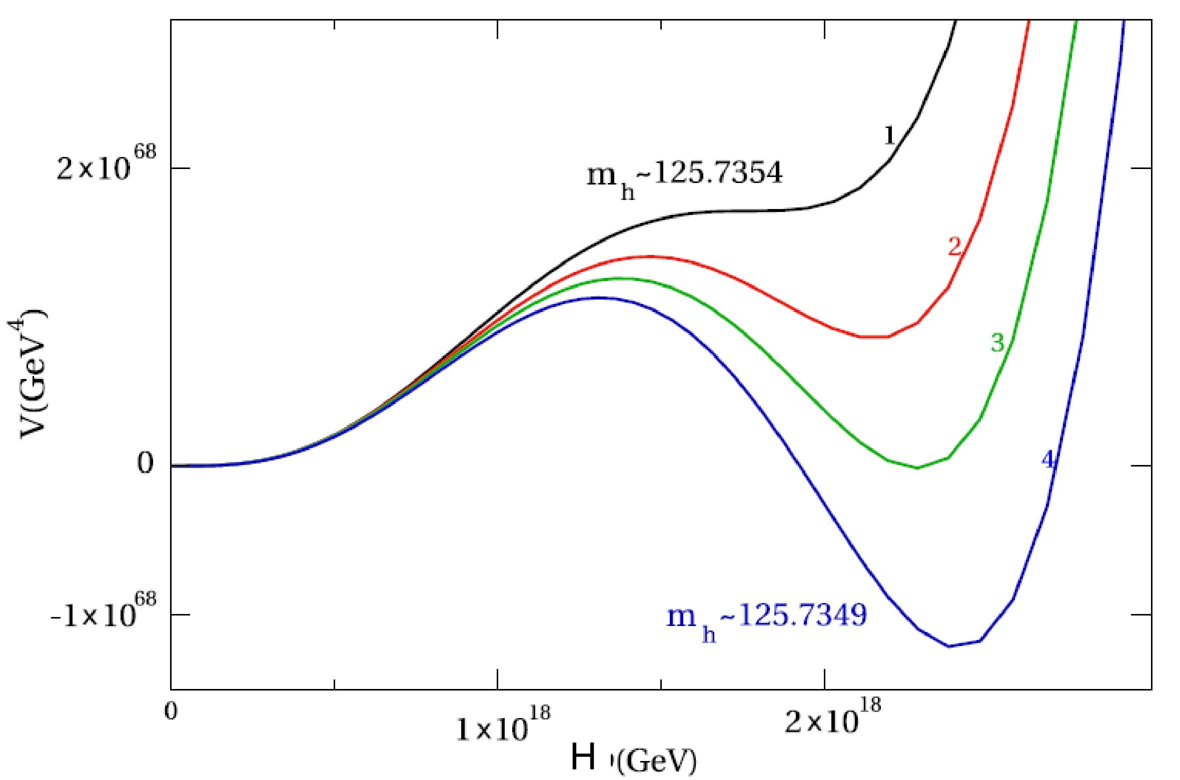
$$\mu_0 = 2\Lambda^2 \frac{f_2}{f_0}$$

$$f_0 = \pi^2/(2g^2)$$

a priori unconstrained

yukawa and majorana parameters subject to RGE aim: flat potential through 2-loop quantum corrections of SM

effective potential at high energies: $V(H) = \lambda(H) H^4$



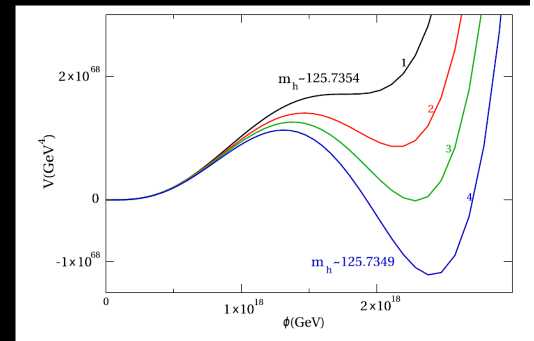
for each value of m_{top} there is a value of m_{higgs} where V_{eff} is on the verge of developing a metastable minimum at large values of **H** and V_{higgs} is locally flattened

approach

- calculate renormalisation of higgs self-coupling
- construct V_{eff} which fits the RG potential around flat region

analytic fit to the higgs potential in the region around the minimum:

$$
\begin{aligned}
V^{\mathrm{eff}} &= \lambda_0^{\mathrm{eff}}(H) H^4 \\
&= [a \ln^2(b\kappa_0 H) + c] H^4
\end{aligned}
$$



$$
\begin{aligned}
a(m_{\mathrm{t}}) &= 4.04704 \times 10^{-3} - 4.41909 \times 10^{-5} \left(\frac{m_{\mathrm{t}}}{\mathrm{GeV}}\right) \\
&\quad + 1.24732 \times 10^{-7} \left(\frac{m_{\mathrm{t}}}{\mathrm{GeV}}\right)^2 \\
b(m_{\mathrm{t}}) &= \exp\left[-0.979261 \left(\frac{m_{\mathrm{t}}}{\mathrm{GeV}} - 172.051\right)\right]
\end{aligned}
$$

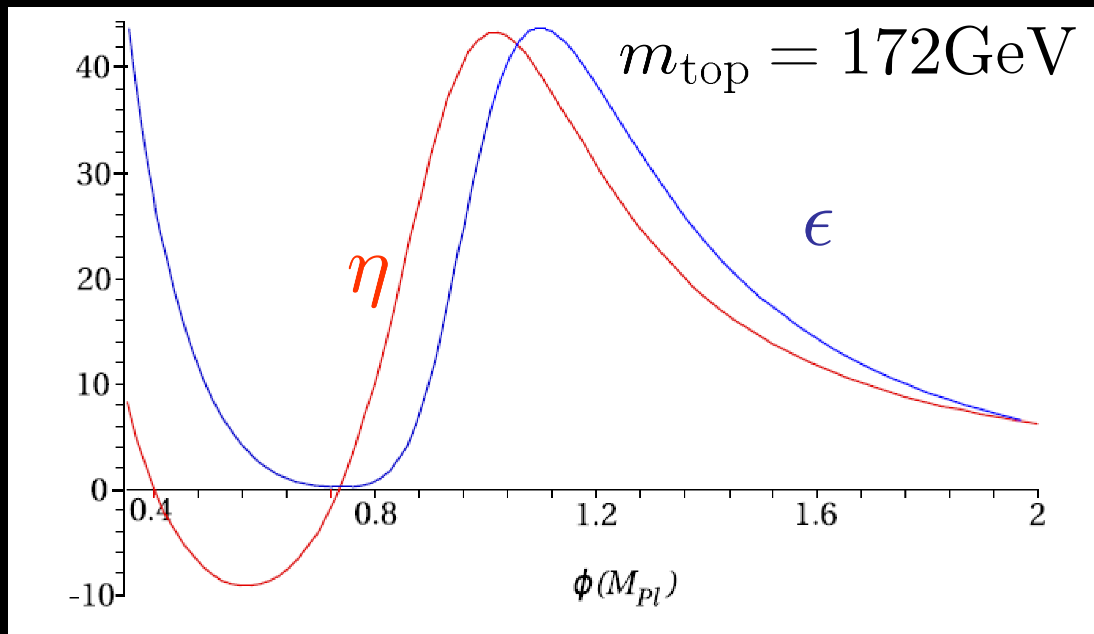
$c = c(m_{\mathrm{t}}, m_\phi)$ encodes the appearance of an extremum

an extremum occurs iff $c/a \leq 1/16$

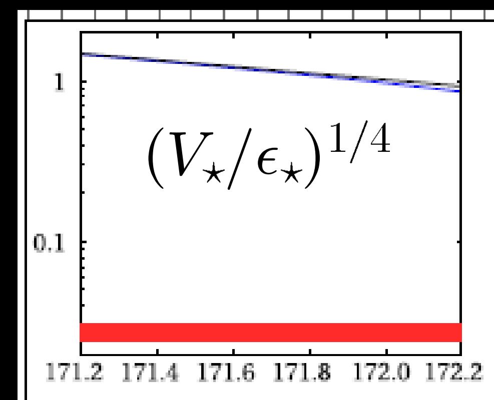
running of the self-coupling at two-loops:

slow-roll conditions satisfied BUT

CMB constraints lead to incompatible top quark mass



$$N \sim \epsilon^{-1/2} \mathrm{d}\phi$$



ϵ needs to be too small to allow for sufficient e-folds, and then $(V_\star/\epsilon_\star)^{1/4}$ becomes too large to fit the CMB constraint

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

can we have inflation without introducing a scalar field?

the arbitrary mass scale in the spectral action for the Dirac operator can be made dynamical by introducing a dilaton field,

$$\mathcal{D}/\Lambda \to e^{-\Phi/2}\mathcal{D}e^{-\Phi/2}$$

$$[redacted] + G^{\mu\nu} D_\mu H'^* D_\nu H' - V_0 \left(H'^* H' \right) \Big] d^4x$$

f: dilaton decay constant

$$\Phi = (1/f)\tilde{\sigma}$$

dilaton

scalar field

could this dilaton field play the rôle of the inflaton?

chamseddine and connes (2006)

criticisms

- simple almost commutative space

  extend to less trivial noncommutative geometries

- purely classical model

  it cannot be used within EU when QC cannot be neglected

- action functional obtained through perturbative approach in inverse powers of cut-off scale

  it ceases to be valid at lower energy scales (astrophysics)

- model developed in euclidean signature

  physical studies must be done in lorentzian signature

the doubling of the algebra is related to dissipation and the gauge field structure

canonical formalism for dissipative systems

x-system: open (dissipating) system

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

$$\frac{d}{dt}\frac{\partial L_f}{\partial \dot{y}} = \frac{\partial L_f}{\partial y}\;; \qquad \frac{d}{dt}\frac{\partial L_f}{\partial \dot{x}} = \frac{\partial L_f}{\partial x}$$

$$L_f(\dot{x},\dot{y},x,y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

$$\Longrightarrow \quad \boxed{m\ddot{x} + \gamma\dot{x} = f\;, \qquad m\ddot{y} - \gamma\dot{y} = 0}$$

{x – y} is a closed system

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation
and the gauge field structure

$$L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{2}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi$$

$$A_i = \frac{B}{2}\epsilon_{ij} x_j \quad (i, j = 1, 2)$$

$$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2)$$

- doubled coordinate, e.g. x_2 acts as gauge field component A_1 to which x_1 coordinate is coupled
- energy dissipated by one system is gained by the other one
- gauge field as *bath/reservoir* in which the system is embedded

sakellariadou, stabile, vitiello , PRD 84 (2011) 045026

dissipation, may lead to a quantum evolution

't hooft's conjecture: loss of information (dissipation) in a regime of deterministic dynamics may lead to QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$H = H_{\mathrm{I}} - H_{\mathrm{II}} \qquad H_{\mathrm{II}}|\psi\rangle = 0 \;\Longrightarrow\; \text{info loss}$$

to define physical states and guarantie that H is bounded from below

physical states are invariant under time reversal and periodical (τ)

$$_H\langle\psi(\tau)|\psi(0)\rangle_H = e^{i\phi} = e^{i\alpha\pi}$$

$$\boxed{\langle\psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega(n + \tfrac{\alpha}{2}) = \hbar\Omega n + E_0}$$

dissipation, may lead to a quantum evolution

dissipation term in H of classical damped-amplified oscillators manifests itself as geometric phase and leads to zero point energy

$$_H\langle\psi(\tau)|\psi(0)\rangle_H = e^{i\phi} = e^{i\alpha\pi}$$

$$\boxed{\langle\psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega(n+\frac{\alpha}{2}) = \hbar\Omega n + E_0}$$

algebra doubling $\longrightarrow$ *deformed hopf algebra*

- define coproduct operators
- build bogogliubov operators as linear combinations of coproduct ones

transformation linking mass annihilation/creation operators with flavor ones is a rotation combined with bogogliubov transformations

$\Downarrow$

field mixing rests on the algebraic structure of the deformed coproduct in NC hopf algebra embedded in algebra doubling of NCSG

gargiulo, sakellariadou, vitiello; arXiv:1305.0659 other cosmological applications?

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}},$$

$$\boxed{\alpha_0 = -\frac{3 f_0}{10\pi^2},}$$

$$\gamma_0 = \frac{1}{\pi^2}\left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d}\right),$$

$$\tau_0 = \frac{11 f_0}{60\pi^2},$$

$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}},$$

$$\xi_0 = \frac{1}{12},$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}.$$

role of scalar fields?

inflation?

given the recent developments of NCSG with the pati-salam model, let me briefly describe issues related to:

- phase transitions associated with spontaneously borken symmetries, leading to topological defect formation as false vacuum remnants
- inflationary models

thermal history of the universe

$\mathrm{G_{SM}} = \mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$

$$\underset{\text{GUT}}{G} \to H_1 \to H_2 \to \cdots \to G_{\rm SM}$$

study homotopy $\pi_k(\mathcal{M}_n)$ group of false vacuum $\mathcal{M}_n = G/H$

$\pi_k(\mathcal{M}_n) \neq 0$ ⇒ topological defects

k=0 ⇒ domain walls

k=1 ⇒ cosmic strings

k=2 ⇒ monopoles

[redacted]

$$G_{\rm GUT} \xrightarrow{M_{\rm GUT}} H_1 \xrightarrow[\Phi_+\Phi_-]{M_{\rm infl}} H_2 \longrightarrow G_{\rm SM}$$

$$
4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ Z_2^{\rm C} \left\{
\begin{array}{lll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ Z_2^{\rm C} & \left\{ \begin{array}{lll} \cdots & & \longrightarrow\ \cdots \\ \xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)}\ {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{2',3\ (2,3)} & {\rm G}_{\rm SM}\ (Z_2) & \end{array} \right. \\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ Z_2^{\rm C} & \left\{ \begin{array}{lll} \xrightarrow{3} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow\ \cdots \\ \xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)}\ {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{3\ (2,3)} & {\rm G}_{\rm SM}\ (Z_2) & \end{array} \right. \\
\xrightarrow{3} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} & \longrightarrow\ \text{Eq. (4.10)} \\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow\ \cdots \\
\xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow\ \cdots \\
\xrightarrow{1,3} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2\ (2)}\ {\rm G}_{\rm SM}\ (Z_2) \\
\xrightarrow{1,3\ (1,2,3)} & {\rm G}_{\rm SM}\ (Z_2). &
\end{array}
\right.
$$

jeannerot, rocher, sakellariadou, PRD 68 (2003) 103514 can we accommodate inflation within minimal SO(10)?

$$SO(10) \rightarrow \cdots \rightarrow G_{3,2,2,B-L} \rightarrow G_{SM} \times Z_2 \rightarrow SU(3)_C \times U(1)_Q \times Z_2\ ,$$
$$SO(10) \rightarrow \cdots \rightarrow G_{3,2,1,B-L} \rightarrow G_{SM} \times Z_2 \rightarrow SU(3)_C \times U(1)_Q \times Z_2$$

$$\tilde{W}_{\mathrm{H}} = m\,\Phi^2 + \lambda\,\Phi^3 + m_H\,H^2 + m_\Sigma\,\Sigma\bar{\Sigma} + \eta\,\Phi\Sigma\bar{\Sigma} + \Phi H\,(\alpha\,\Sigma + \bar{\alpha}\,\bar{\Sigma}) + m_\Omega\,\Omega^2 + \beta\,H\Phi\Omega + \gamma\,\Omega^2\Phi + \Omega\Phi(\zeta\,\Sigma + \bar{\zeta}\,\bar{\Sigma})\ .$$

none of the singlets of SM symmetries in minimal set of SO(10) rep. can satisfy conditions for scalar field to be inflaton

cacciapaglia, sakellariadou, arXiv:1306.3242

final remarks/questions

- what are the new fields in the pati-salam model within the NCSG approach?

- what can we say for inflation? can any of the new fields play the role of the inflaton?

- can we accommodate inflation at the last stage of phase transition accompanied bt SSB during which unwanted defects get formed?