Hidden Causal Structure of Gauge Theories

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with Nicolas Franco (Namur) & Tomasz Miller (MiNI PW, CC)

Jagellonian University & Copernicus Center, Kraków, Poland

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Banach–Simons Sem 1 Sep – 30 Nov 2016, IMPAN, PL

Banach-Simons Semester

Banach Center
INSTITUTE OF MATHEMATICS PAS

SIMONS FOUNDATION

1 Sep – 30 Nov 2016, Simons Semester in the Banach Center

NONCOMMUTATIVE GEOMETRY THE NEXT GENERATION

Paul F. Baum, Alan Carey, Piotr M. Hajac, Tomasz Maszczyk

Funding available for longer stays (Senior Professors and Junior Professors, Postdocs, or PhD Students).

Banach–Simons Sem 1 Sep – 30 Nov 2016, IMPAN, PL

Noncommutative Geometry the Next Generation

1. 4–17 September, Będlewo & Warsaw, school on
   Noncommutative geometry and quantum groups
2. 19 September – 14 October, 20-hour lecture course
   An invitation to C*-algebras by Karen R. Strung
3. 19 September – 14 October, 20-hour lecture course
   An invitation to Hopf algebras by Réamonn Ó Buachalla
4. 19 September – 14 October, 20-hour lecture course
   Noncommutative topology for beginners by Tatiana Shulman
5. 17–21 Oct. Cyclic homology
   J. Cuntz, P. M. Hajac, T. Maszczyk, R. Nest
6. 24–28 Oct. Noncommutative index theory
   P. F. Baum, A. Carey, M. J. Pflaum, A. Sitarz
7. 14–18 Nov. Topological quantum groups and Hopf algebras
   K. De Commer, P. M. Hajac, R. Ó Buachalla, A. Skalski
8. 21–25 Nov. Structure and classification of C*-algebras
   G. Elliott, K. R. Strung, W. Winter, J. Zacharias

Is the World noncommutative?

- NCG is a convenient tool to generate particle physics models.
  - pros: constraints on admissible models, gravity included, concrete predictions
  - cons: classical fields, wrong signature
- Is NCG *only* a tool?
- Common viewpoint: almost commutative geometry is an approximation of a *truly noncommutative spacetime*.
- Conceptual problems related:
  - What is space(time), interaction, evolution, symmetry, ...
- What are the empirical signatures of the NCG?

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Outline

1. Noncommutative geometry à la Connes
2. Lorentzian aspects of NCG
3. Causal structure of gauge theories
4. Is there a hidden causal structure of gauge theories?

Spectral triples

$(\mathcal{A}, \mathcal{H}, \mathcal{D})$ – spectral triple

- $\mathcal{A}$ – dense $*$-subalgebra of a C^*-algebra (unital).
- $\mathcal{H}$ – Hilbert space with a faithful representation $\rho(\mathcal{A}) \subset \mathcal{B}(\mathcal{H})$.
- $\mathcal{D}$ – a Dirac operator – densely defined on $\mathcal{H}$, selfadjoint,
    - $(\mathcal{D} - \lambda)^{-1}$ for any $\lambda \notin \mathbb{R}$ – compact resolvent,
    - $[\mathcal{D}, \rho(a)] \in \mathcal{B}(\mathcal{H})$ for all $a \in \mathcal{A}$.
- $+$ additional structure – real structure J, chirality
- $+$ technical assumptions – smoothness, finite dimensionality
- $+$ other assumptions – first order condition, ...

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Almost commutative manifolds

Connes' Reconstruction Theorem [1996–2008]

For every *commutative* spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ there exists a smooth compact spin Riemannian manifold M such that:

$$\mathcal{A} = C^\infty(M), \qquad \mathcal{H} = L^2(S(M)), \qquad \mathcal{D} = \not{D} = -\,\mathrm{i}\,\gamma^\mu \nabla^S_\mu.$$

- Finite ST: $\mathcal{A}_\mathcal{F} = \bigoplus M_n(\mathbb{C})$, $\mathcal{H}_\mathcal{F} = \mathbb{C}^N$, $\mathcal{D}_\mathcal{F} = \mathcal{D}_\mathcal{F}^\dagger \in M_N(\mathbb{C})$.
- Almost commutative geometry:
  - $\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_\mathcal{F}$ (gauge group)
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NC Standard Model

1. Take: $\mathcal{A}_{\mathcal{F}} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ and suitable $\mathcal{H}_{\mathcal{F}}$, $\mathcal{D}_{\mathcal{F}}$ ...
2. ... compute ...

$$S = S_F + S_B = \langle J\psi | \mathcal{D}\psi \rangle + \mathrm{Tr}\left(f(\mathcal{D}/\Lambda) \right) = \int_M \mathcal{L}$$

3. ... forget about NCG and ...
4. ... quantise, Wick-rotate, launch the RG flow, ...
5. ... and ...
   - recover the Standard Model,
   - predict new fields and masses,
   - address cosmological questions.

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Connes' distance formula

- States $S(\mathcal{A}) = \{\varphi\}$ on a (pre-)C^*-algebra $\mathcal{A}$:
  - positive linear functionals with $\|\varphi\| = 1$.
  - $S(\mathcal{A})$ is a closed convex set (for the weak-* topology).
  - $P(\mathcal{A})$ – extremal points of $S(\mathcal{A})$ – pure states.

Connes' (pseudo-)distance formula

For $\chi, \xi \in S(\mathcal{A})$

$$d(\chi, \xi) = \sup_{a \in \mathcal{A}} \{ |\chi(a) - \xi(a)| : \ \|[\mathcal{D}, a]\| \leq 1 \}$$

- For $A = C_0(M)$, $P(A) \simeq M$, $S(A) \simeq \mathfrak{P}(M)$.

$$d(\varphi_p, \varphi_q) = d_{\text{geo}}(p, q), \qquad d(\mu, \nu) = W_1(\mu, \nu)$$

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Connes' distance formula

- States $S(\mathcal{A}) = \{\varphi\}$ on a (pre-)C^*-algebra $\mathcal{A}$:
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Internal spaces

- From the spectral point of view $(\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, \mathcal{D}_{\mathcal{F}})$ is 0-dimensional.

States on finite algebras

Let $\mathcal{A} = M_n(\mathbb{C})$, then all of the pure states are vector states, $P(\mathcal{A}) \simeq \mathbb{C}P^{n-1}$ – 'qunits' or 'n-qubits'. Mixed states correspond to density matrices.

- $P(\mathcal{A}_1 \oplus \mathcal{A}_2) = P(\mathcal{A}_1) \sqcup P(\mathcal{A}_2)$

Theorem [Kadison (1986)]

If at least one of the C^*-algebras $\mathcal{A}_1$, $\mathcal{A}_2$ is commutative, then $P(\mathcal{A}_1 \otimes \mathcal{A}_2) \cong P(\mathcal{A}_1) \times P(\mathcal{A}_2)$, *i.e.* pure states on $\mathcal{A}_1 \otimes \mathcal{A}_2$ are separable.

- Distances of finite spaces [D'Andrea, Iochum, Krajewski, Martinetti, . . .]

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Pseudo-Riemannian spectral triple

- The machinery of $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ suited to Riemannian manifolds.
- Need for indefinite products $\rightsquigarrow$ Krein spaces
- 'Work in progress'
  Bizi, van den Dungen, Franco, Pashke, Rennie, Sitarz, Strohmaier, . . .

Pseudo-Riemannian spectral triple – a 'minimal' definition

$\mathcal{A}$ is a dense $*$-subalgebra of a C^*-algebra faithfully represented on a Krein space $\mathcal{K}$ and $\mathcal{D}$ is a densely defined Krein-symmetric operator on $\mathcal{K}$ such that $[\mathcal{D}, a]$ extends to a bounded operator for all $a \in \mathcal{A}$.

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The causal structure

Choose a suitable *unitisation* of $\mathcal{A}$, $\widetilde{\mathcal{A}}$ (technicality).

A causal cone $\mathcal{C} \subset \widetilde{\mathcal{A}}$ is such that $\forall_{a,b \in \mathcal{C}}$

(0) $a = a^*, \quad a + b \in \mathcal{C}, \quad \forall_{\lambda \geq 0} \ \lambda a \in \mathcal{C}, \quad \forall_{x \in \mathbb{R}} \ x1 \in \mathcal{C};$

(1) $\forall_{a \in \mathcal{C}} \forall_{\phi \in \mathcal{K}} \quad (\phi, [\mathcal{D}, a]\phi) \leq 0;$

(2) $\overline{\mathrm{span}_{\mathbb{C}}(\mathcal{C})} = \overline{\widetilde{\mathcal{A}}}.$

Proposition [N. Franco, M.E. (2013)]

Let $\mathcal{C} \subset \widetilde{\mathcal{A}}$ be a causal cone, then for every two states $\chi, \xi \in S(\widetilde{\mathcal{A}})$ define

$$\chi \preceq \xi \quad \text{iff} \quad \forall_{a \in \mathcal{C}} \quad \chi(a) \leq \xi(a).$$

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Classical picture

'Bottom-up' reconstruction

If M is a pseudo-Riemannian manifold, then $\mathcal{A}_M = C_c^\infty(M)$, $\mathcal{K}_M = L^2(M,S)$, $\mathcal{D}_M = \not{D}$ constitutes a pseudo-Riemannian ST.

Theorem [N. Franco, M.E. (2013)]

Let M be a causally simple spacetime and $(\mathcal{A},\mathcal{K},\mathcal{D})$ the associated pseudo-Riemannian ST, then $P(\mathcal{A}) \simeq M$ and

$$\varphi_p \preceq \varphi_q \quad \text{if and only if} \quad p \preceq q \text{ on } M.$$

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Almost-commutative spacetimes

Let $(\mathcal{A}_M, \mathcal{K}_M, \mathcal{D}_M)$ be an even ($\mathcal{K}$ is $\mathbb{Z}_2$-graded), then $\mathcal{A}_M \otimes \mathcal{A}_{\mathcal{F}}$, $\mathcal{K}_M \otimes \mathcal{H}_{\mathcal{F}}$, $\mathcal{D}_M \otimes 1 + \gamma^5 \otimes \mathcal{D}_{\mathcal{F}}$ is an almost commutative pseudo-Riemannian spectral triple.

Almost-commutative spacetime

$$P(\mathcal{A}) = M \times P(\mathcal{A}_{\mathcal{F}})$$

Theorem (No Einstein causality violation) [M.E., Franco (2014b)]

Let $(\mathcal{A}, \mathcal{K}, \mathcal{D})$ be an almost commutative pseudo-Riemannian spectral triple, such that the causal cone $\mathcal{C}$ exists. If $\omega_{p,\xi}, \omega_{q,\chi} \in P(\mathcal{A})$ are such that $\omega_{p,\xi} \preceq \omega_{q,\chi}$, then $p \preceq q$ in M.

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Two-sheeted space-time

- $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{C}$,
- $\mathcal{H}_F = \mathbb{C}^2$,
- $\mathcal{D}_F = \left(\begin{smallmatrix} 0 & m \\ m^* & 0 \end{smallmatrix}\right)$.
- $P(\mathcal{A}) = M \times \{0,1\} = M \sqcup M$.

Theorem [*N. Franco, M.E. 2015b*]

$(p,0) \preceq (q,1)$ iff $p \preceq q$ in M and

$$\tau(\gamma) \geq \frac{\pi}{2\,|m|},$$

where $\tau(\gamma)$ – proper time along γ.

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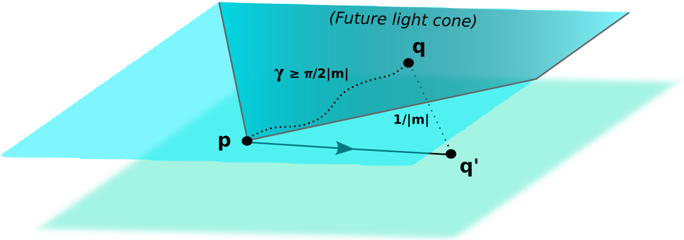

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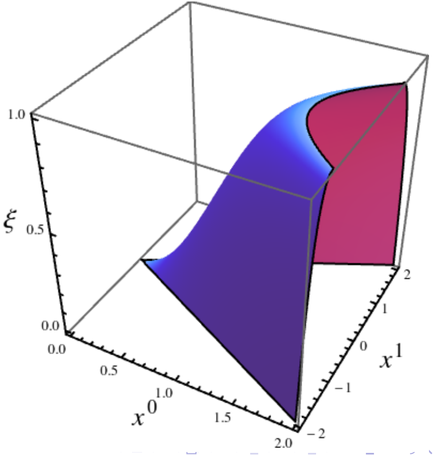

Zitterbewegung

- Zitterbewegung – the 'trembling motion of the electron'.
- Free Dirac equation $i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$.
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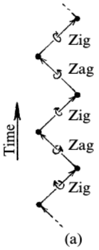

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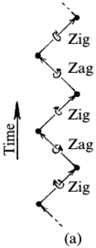

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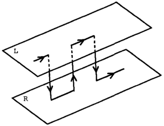

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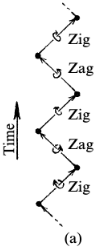

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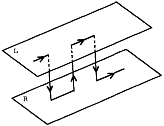

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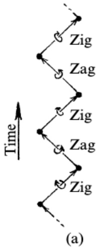

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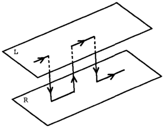

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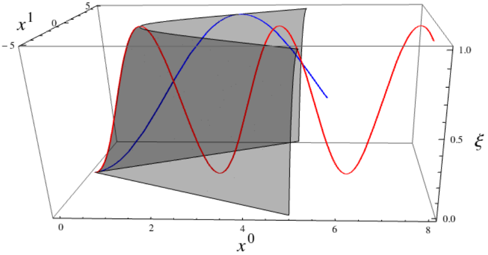

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- Fluctuations of Dirac operator: $\mathcal{D} \rightsquigarrow \mathcal{D}_{\mathbb{A}} = (\not{D} - \not{A}\,) \otimes 1 + \gamma_M \otimes \Phi$.
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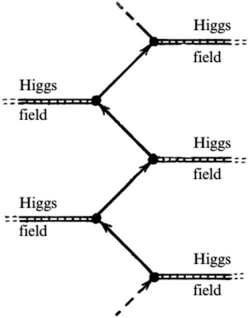

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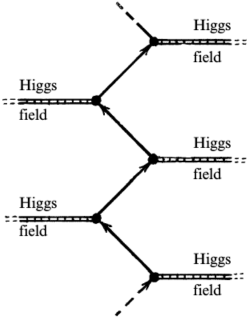

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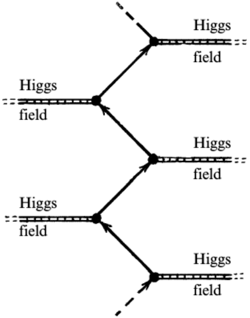

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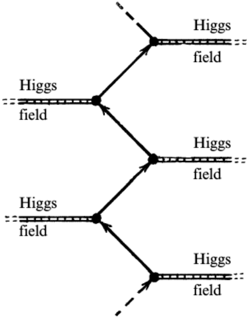

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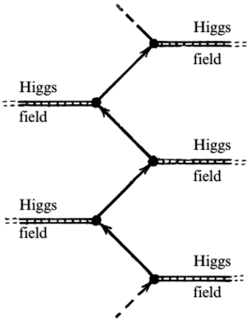

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- $\mathcal{A} = C_c^\infty(M)$, $S(\mathcal{A}) = \mathfrak{P}(M)$ – probability measures on M.
- When does $\mu \preceq \nu$? [M.E., T. Miller (2015)]
- A link with optimal transport theory.
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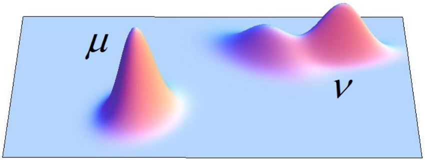

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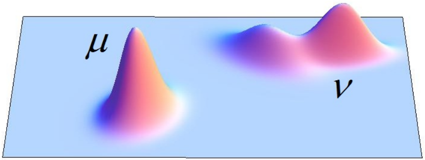

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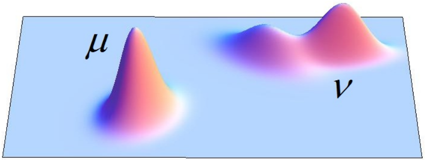

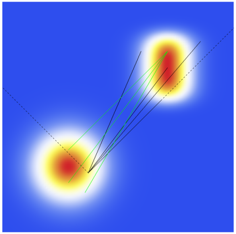

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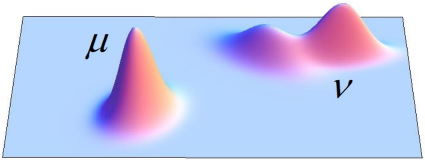

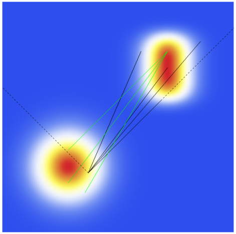

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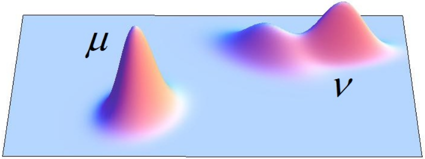

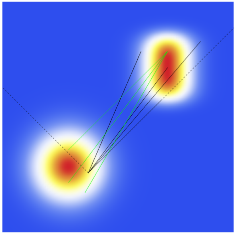

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Evolution in QM

- Wave packet formalism on $(n+1)$-dim Minkowski spacetime.
- The Schrödinger equation $i\hbar\partial_t\psi = \hat{H}\psi$.
- Any wavefunction $\psi \in L^2(\mathbb{R}^{n+1})$ yields a family of measures on M

$$\{\mu_t \in \mathfrak{P}(\mathbb{R}^{n+1})\}_t, \qquad \mu_t = \delta_t \times \|\psi(t,x)\|^2 \, d^n x.$$

- Is the quantum evolution causal, i.e. $\mu_s \preceq \mu_t$ if $s < t$?

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Dirac equation yields a causal evolution!

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- In the Dirac equation

$$\psi(t,x) = \psi^L(t,x) + \psi^R(t,x), \qquad \mu_t = \mu_t^L + \mu_t^R.$$

Conjecture

Dirac equation respects causality on the *two-sheeted spacetime*.

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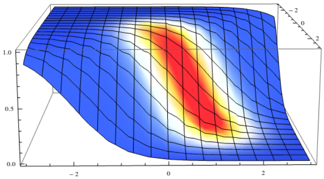

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Conjecture

Dirac equation respects causality on the *two-sheeted spacetime*.

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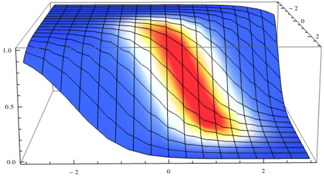

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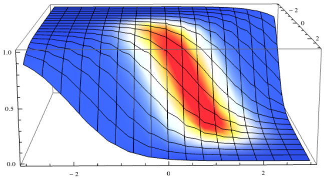

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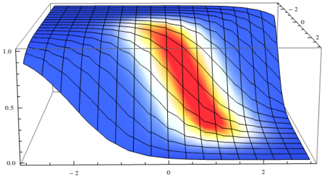

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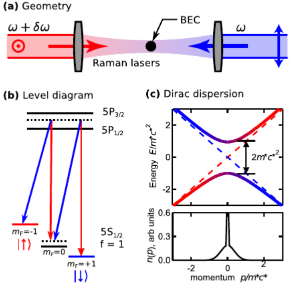

L.J. LeBlanc et al., New J. of Phys. **15** 073011 (2013)

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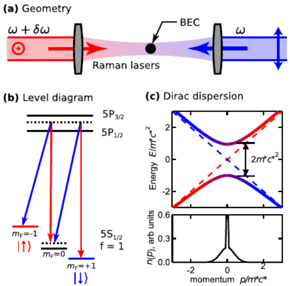

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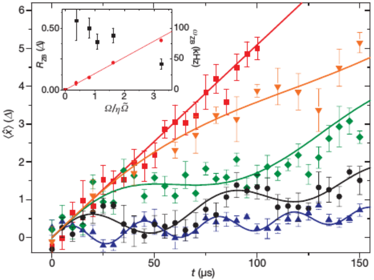

R. Gerritsma et al., Nature **463** 68 (2010)

Is there a hidden causal structure of gauge theories?

- There is a non-trivial *Lorentzian* geometry on $M \sqcup M$.
- *Abelian* gauge field does not affect it, but the Higgs does.
- $P(\mathbb{C} \oplus \mathbb{H}) = P(\mathbb{C} \oplus \mathbb{C})$, but there is much more ...
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- Micro-causality condition: $[A(x), A(y)] = 0$ whenever $x \not\preceq y$.
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- $\Rightarrow$ we should have $[A_L(x), A_R(y)] = 0$ whenever $(x, L) \not\preceq (y, R)$.

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The bibliography

Thank you for your attention!

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