The Gribov problem in Noncommutative QED

Patrizia Vitale

Dipartimento di Fisica Università di Napoli "Federico II" and INFN

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Outline

- The Gribov problem in standard gauge theory

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- Perspectives

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Euclidean action for gauge fileds

$$S[A] = \frac{1}{4} Tr \int F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} \int A_\mu^a M^{\mu\nu} A_\nu^a$$

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not invertible because of gauge invariance $\quad g(x) = \exp \alpha(x)$

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$(A_\mu^g = A_\mu + \partial_\mu \alpha$, $\partial_\mu \alpha$ is a zero mode).

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This amounts to choose a surface $\Sigma_f \subset \mathcal{A}$ which possibly intersects the gauge orbits only once: a section for the principal bundle

$$\begin{array}{ccc} \mathcal{A} & \leftarrow & \mathcal{G} \\ \downarrow & & \\ \mathcal{B} & & \end{array}$$

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To perform the change of variable $[d\alpha] \to [df^a(A)]$:
insert the Jacobian

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and integrate over [df] with the delta function:

$$[d\mu(\mathcal{B})] = [d\mu(\mathcal{A})]\ \mathrm{Det}\Delta\ \ \delta(f(A) - h(x))$$

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for $G = U(N)$ $\Pi_5 = Z$, $N \geq 3$; $\Pi_5 = Z_2$, $N = 2$; $\Pi_5 = 0$, $N = 1$

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Summary: Non-Abelian gauge theories do not admit global sections

This amounts to the FP operator Δ having non trivial zero modes (the determinant of the Jacobian changes its sign when the surface of gauge fixing meets the gauge orbits more than once).

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$$G^{GZ}(p) \simeq \frac{p^2}{p^4 + a^4}$$

The Moyal algebra $\mathbb{R}_\theta^{2n}$

- It is the associative algebra [J. Gracia Bondia, Varilly '89]

$$\mathcal{L} \cap \mathcal{R} = \{T \in \mathcal{S}' : T \star f \in \mathcal{S}, f \star T \in \mathcal{S}, \forall f \in \mathcal{S}\}$$

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The star product is defined for Schwartz functions on $\mathbb{R}^{2n}$

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and extended to tempered distributions by duality.
Θ is block diagonal, antisymmetric with θ_i real.

$$\Theta = \begin{pmatrix} 0 & -\theta_1 & \\ \theta_1 & 0 & \\ & & .. \end{pmatrix}$$

with these defs R_θ^{2n} is unital and involutive. It contains $\mathcal{S}$ and polynomials. Constants are in the center.

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- Vector bundles are replaced by right modules over the algebra $\mathbb{R}^{2n}_\theta$, with Hermitian structure h

$$h(m_1 \star f_1, m_2 \star f_2) = f_1^\dagger \star h(m_1, m_2) \star f_2, \quad f_i \in \mathbb{R}^{2n}_\theta, m_i \in \mathcal{H}$$

The connection is defined as

$$\nabla : Der(\mathbb{R}_\theta^{2n}) \times \mathcal{H} \to \mathcal{H}, \quad \nabla_\mu(m \star f) = \nabla_\mu m \star f + m \star \partial_\mu f$$

preserving the Hermitian structure. It is completely defined by the action on a basis of $\mathcal{H}$.

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We want to generalize the $U(1)$ gauge connection
U(1) vector bundle is replaced by the right module (one generator)

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- The gauge connection is defined by its action on the basis

$$\nabla_\mu(\mathbf{1}) \equiv -iA(\partial_\mu)$$

so that $\quad \nabla_\mu f = \nabla_\mu(\mathbf{1} \star f) = \partial_\mu f - iA_\mu \star f$

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Properties of the gauge connection

- $(\nabla^A_\mu)^\gamma(\phi) := \gamma(\nabla^A_\mu(\gamma^{-1}\phi)) = U \star \nabla^A_\mu U^{-1} \star \phi$
- $A^U_\mu = U \star A_\mu \star U^{-1} + iU \star \partial_\mu U^{-1}$
- $F_{\mu\nu} = i([\nabla^A_\mu, \nabla^A_\nu] - \nabla^A_{[X_\mu, X_\nu]}) = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_\star$
- $F^U_{\mu\nu} = U \star F_{\mu\nu} \star U^{-1}$

Noncommutative QED on R_θ^{2n}

The natural QED action is gauge and Poincaré invariant but yields new pathologies w.r.t. the commutative case (UV/IR mixing)

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Asymptotycally:

$$(f \star g)(x) = f(x) \exp\left\{ \frac{i}{2} \theta^{\rho\sigma} \overleftarrow{\partial}_\rho \overrightarrow{\partial}_\sigma \right\} g(x)$$

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Under the $U(1)$ gauge transformation in NCQED the gauge field A transforms as

$$A \to A'_\mu[\alpha] = U \star A_\mu \star U^\dagger + i\, U \star \partial_\mu U^\dagger, \qquad U \equiv \exp_\star (i\alpha)$$

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Fourier transforming we get a homogeneous Fredholm equation of second kind

$$\hat{\alpha}(k) = \int d^d q \; Q(q,k) \; \hat{\alpha}(q)$$

with the kernel Q given by

$$Q(q,k) = -\frac{2i\, k^\mu \hat{A}_\mu(k-q)}{k^2} \sin\left(\frac{1}{2}\,\theta^{\rho\sigma} q_\rho k_\sigma\right)$$

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The existence of Gribov copies has been recast into an eigenvalue equation for the operator Q. [properties]

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Unfortunately, this connection is gauge invariant: a fixed point of the gauge group $\to$ no new copies.

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  - in Fourier transform $A_\mu(x) \propto \epsilon_{\mu\nu} \Box \partial^\nu \delta(k)$
  - zero modes equation:

$$\left(-4k^2 \Box - 8\, \varepsilon^{\mu\nu} \varepsilon^{\eta\lambda} k_\mu k_\eta \partial_\nu \partial_\lambda - \frac{4k^2}{Q\theta} + \theta^2 k^4 \right) \hat{\alpha}(k) = 0$$

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infinite number of solutions in terms of special functions.

$$\hat{\alpha}_{nm}(r,\phi) = (C_1 \cos n\phi + C_2 \sin n\phi) r^{\sqrt{3n^2+1}-1} \exp(-\frac{r^2\theta}{4}) L_m^{\sqrt{3n^2+1}}(\frac{\theta r^2}{2})$$

- Next to the simplest situation
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can be extended to 4d case.

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  - Compute $S[\phi_U] - S[\phi]$ and study the equation of "copies" and the correction to the propagator.
  - Compare with the translation invariant scalar model Gurau, Magnen, Rivasseau, Tanasa '09

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Bibliography

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Bibliography

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D. N. Blaschke, "Aspects of perturbative quantum field theory on non-commutative spaces," arXiv:1601.03109 [hep-th].

R. Gurau, J. Magnen, V. Rivasseau and A. Tanasa, "A Translation-invariant renormalizable non-commutative scalar model," Commun. Math. Phys. **287**, 275 (2009) [arXiv:0802.0791 [math-ph]].