Structure of the Nucleon and Geometry of the Wilson Loop Space

Igor O. Cherednikov

Antwerpen & Dubna

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What we can learn from the study of Wilson loops?

- Emergence of the generalized Wilson loop space in the quantum field-theoretical description of the 3D-structure of the nucleon visible in high-energy hadron collisions
- Geometrical properties of the loop space can be used for understanding of the most general properties of the nonperturbative distribution of partons inside the nucleon
- Duality between equations of motion in the loop space and evolution of the 3D-parton densities

Ancient Greece

Aristotle: three divisions of human intellectual activity

- **physics**: studies the causes of change of material things
- **mathematics**: addresses abstract quantity
- **metaphysics**: concerned with being as such

Ergo: physics is the learning of **evolution**

Many Faces of QCD Evolution

Renaissance: **3D structure of the nucleon**

- Experiment:
  - **SIDIS process** $lH^{\uparrow} \to l' hX$**:** HERMES, COMPASS
  - **DY process** $H_1^{(\uparrow)} H_2^{\uparrow} \to l^+ l^- X$ **:** COMPASS, PAX, GSI, RHIC
  - **Hadron collisions** $H_1^{(\uparrow)} H_2^{\uparrow} \to l^+ l^- X$ **:** RHIC
  - $e^+ e^- \to h_1 h_2 X$**:** BELLE, BaBar
  - future **EIC, JLab 12 GeV**
- Phenomenology: single-spin asymmetries; polarized and non-polarized parton distributions; etc.
- Theory: transverse-momentum dependent quark and gluon densities

Many Faces of QCD Evolution

- Experiment
- Phenomenology
- Theory
- Mathematics: **Loop Space** analysis
  - Chen iterated line integrals (via the holonomy of a connection in a principal fiber bundle)
  - Group of loops and generalised group of loops
  - Loop calculus: endpoint and area derivatives; variational derivative
  - Connection with knot theory *etc.*

@ [Chen (1968, 1971, 1973); Gambini, Trias (1980, 1981, 1986); Tavares (1993)]

3D Tomography of the Nucleon
Prerequisites

- High-energy electron-nucleon scattering experiments: **one-dimensional structure** of the protons and neutrons; **collinear parton distribution functions** (PDFs)—probability to find a parton (quark or gluon) with certain momentum inside the nucleon: **well-defined gauge-invariant universal** nonperturbative objects; evolution is known (see DGLAP)
- Semi-inclusive reactions with polarized and unpolarized hadrons; new era in exploring the structure of nucleons **beyond the collinear approximation**: **(sometimes) ill-defined (not fully) gauge-invariant (often non-) universal** nonperturbative objects; evolution is under discussion
- Collinear PDFs $\to$ **transverse-momentum dependent parton densities** (TMDs): rather non-trivial generalisation; very complicated structure of the **Wilson lines**

What are the Wilson Lines/Loops?
Gauge-Invariant Hadronic Correlators

$$\mathcal{F}(k) = \mathrm{F.T.}\ \langle h|\ \bar{\Psi}(z)\ \mathcal{W}_\Gamma[z,0]\ \Psi(0)\ |h\rangle$$

Gauge invariance is guaranteed by the **Wilson line**, or the **gauge link**, or the **eikonal line**

$$\mathcal{W}_\Gamma = \mathcal{P}\ \exp\left[-ig\int_\Gamma d\zeta^\mu \mathcal{A}_\mu(\zeta)\right]$$

- Gauge invariance
- Path dependence and universality
- Singularities and renormalization
- Evolution and factorization

3D Hadronic Correlators: Structure of Nucleon beyond the Collinear Approximation

@ [Belitsky, Ji, Yuan (2003); Boer, Mulders, Pijlman (2003)]

Generic 3D hadronic correlator with the light-like and transverse gauge links

$$\mathcal{F}(k^+, k_\perp; \text{scales}) \sim$$

$$\text{F.T.} \ \langle h|\bar{\Psi}(z) \ \mathcal{W}_{n\cup l_\perp}[z^-, z_\perp; 0^-, 0_\perp]\Psi(0)|h\rangle$$

Tree-level:

$$\mathcal{F}^{(0)}(k^+, k_\perp) = \delta(k^+ - p^+)\delta^{(2)}(k_\perp)$$

$$\int d^2k_\perp \Phi(k^+, k_\perp) = \mathcal{F}(k^+) = \text{collinear limit}$$

$$\mathcal{F}(k^+, \mu) = \int dz^- \ \mathrm{e}^{-ik^+z^-} \langle h|\bar{\Psi}(z) \ \mathcal{W}_n[z^-, 0^-]\Psi(0)|h\rangle$$

Quantum corrections: $\rightarrow$ emergent (light-cone/rapidity/overlapping) singularities $\rightarrow$ problems with renormalization and evolution

Classification of Singularities

- Ultraviolet poles $\sim \frac{1}{\varepsilon}$
- Overlapping divergences: contain the UV and rapidity poles simultaneously $\sim \frac{1}{\varepsilon} \ln \theta$
- Pure rapidity divergences: $\sim \ln^{1,2} \theta$:
- Specific self-energy divergences: stem from the gauge links, treated by modifications of the soft factors

@ [ICh, Stefanis (2008, 2009, 2010); Collins (2003, 2008, 2011, 2012 etc.); Chiu, Jain, Neill, Rothstein (2011, 2012); Avsar (2012) Idilbi, Scimemi (2011, 2012)]

- Penetration of the extra singularities in the anomalous dimensions of the TMDs:

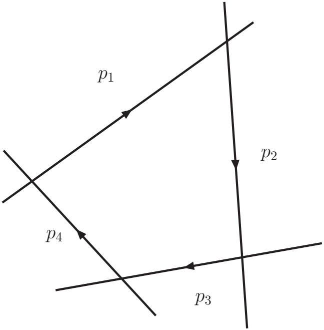
@ [ICh, Stefanis (2008, 2009, 2010)]

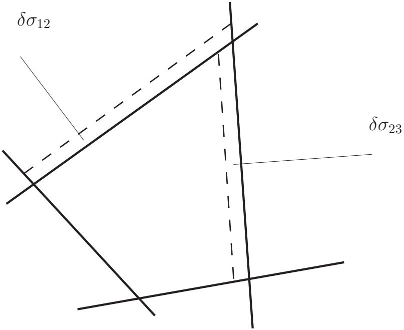
- Collinear case: cancellation in the interplay of the virtual and real gluon contributions:

@ [Furmanski, Curci, Petronzio (1980); Fleming, Zhang (2012)]

Singularities of Light-Like Cusped Wilson Loops

Generic light-like quadrilateral contour





Singularities of Light-like Cusped Wilson Loops
Generic light-like quadrilateral contour

@ [Alday, Maldacena (2007); Makeenko (2003); Korchemsky, Drummond, Sokatchev (2008); Alday et al. (2011); Beisert et al. (2012); Belitsky (2012)]

Motivation: **duality** between 4-gluon planar scattering amplitude in $\mathcal{N} = 4$ SYM and the Wilson loop made up from four light-like segments:

$$x_i - x_{i+1} = p_i$$

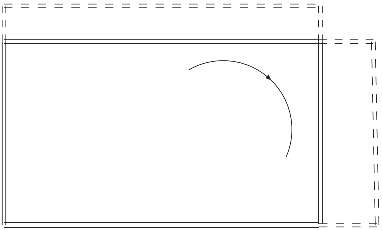
are equal to the external momenta of this 4-gluon amplitude. The IR evolution of the former is dual to the UV evolution of the latter: governed by the cusp anomalous dimension.

@ [Korchemsky, Radyushkin (1987)]

Singularities of Light-like Cusped Wilson Loops

Planar light-like rectangular Wilson loop: a "Hydrogen Atom" of the WL analysis

@ [Korchemskaya, Korchemsky (1992); Bassetto, Korchemskaya, Korchemsky, Nardelli (1993)]



$$\sigma \equiv 2N^+N^- \ , \ \lim_{N_c \to \infty} \mathcal{W}[\Gamma] =$$

$$1 + \frac{\alpha_s N_c}{2\pi} \left\{ - \frac{1}{\epsilon^2} \left(\left[-\sigma\mu^2 + i0 \right]^\epsilon + \left[\sigma\mu^2 + i0 \right]^\epsilon \right) + \text{finite} \right\} + O(\alpha_s N_c)$$

Singularities of Light-like Cusped Wilson Loops

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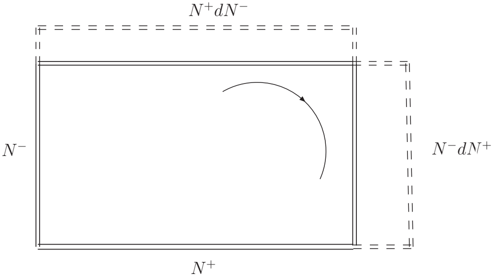


– $\mathcal{W}[\Gamma]$ is not renormalizable due to light-cone extra divergences
– Aristotle: physics concerns with evolution, not the Wilson loops as such
– area logarithmic derivative decreases the power of divergency

$$\frac{d \ln \mathcal{W}[\Gamma]}{d \ln \sigma} = -\frac{\alpha_s N_c}{2\pi} \frac{1}{\epsilon} \left([\sigma \mu^2 + i0]^\epsilon - [-\sigma \mu^2 + i0]^\epsilon \right)$$

Conclusions I:

- Wilson Loop Space emerges naturally in the quantum field-theoretical description of the 3D-structure of the nucleon
- Singularities of the Wilson Loops open the door to the analysis of renormalization and evolution of the gauge invariant correlation functions
- We must understand the structure of the WLS in detail

Mathematics: Loop Space

@ [Polyakov (1979); Makeenko, Migdal (1979, 1981); Kazakov, Kostov (1980); Brandt et al. (1981, 1982); Stefanis et al. (1989, 2003)]

Wilson loops as the (fundamental) gauge-invariant degrees of freedom:

$$\mathcal{W}_n[\Gamma_1, ...\Gamma_n] = \langle 0|\mathcal{T}\frac{1}{N_c}\Phi(\Gamma_1)\cdots\frac{1}{N_c}\Phi(\Gamma_n)|0\rangle$$

$$\Phi(\Gamma_i) = \mathcal{P}\ \exp\left[ig\int_{\Gamma_i} dz^\mu \mathcal{A}_\mu(z)\right]$$

The Wilson functionals obey the Makeenko-Migdal loop equations:

$$\partial^\nu\ \frac{\delta}{\delta\sigma_{\mu\nu}(x)}\ \mathcal{W}_1[\Gamma] = N_c g^2\ \oint_\Gamma dz^\mu\ \delta^{(4)}(x-z)\mathcal{W}_2[\Gamma_{xz}\Gamma_{zx}]$$

$+$ Mandelstam contraints

$$\sum a_i \mathcal{W}_{n_i}(C_1^i ... C_{n_i}^i) = 0$$

The equation is exact...

Area derivative:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\Phi(\Gamma) = \lim_{|\delta\sigma_{\mu\nu}(x)|\to 0} \frac{\Phi(\Gamma\delta\Gamma_x) - \Phi(\Gamma)}{|\delta\sigma_{\mu\nu}(x)|}$$

Path derivative:

$$\partial_\mu\Phi(\Gamma) = \lim_{|\delta x_\mu|\to 0} \frac{\Phi(\delta x_\mu^{-1}\Gamma\delta x_\mu) - \Phi(\Gamma)}{|\delta x_\mu|}$$

Mandelstam formula:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\mathrm{Tr}\ \Phi(\Gamma) = ig\mathrm{Tr}\ [F_{\mu\nu}\Phi(\Gamma)]$$

$$\partial^\nu \, \frac{\delta}{\delta\sigma_{\mu\nu}(x)} \, \mathcal{W}_1[\Gamma] = N_c g^2 \, \oint_\Gamma \, dz^\mu \, \delta^{(4)}(x-z)\mathcal{W}_2[\Gamma_{xz}\Gamma_{zx}]$$

The equation is exact and non-perturbative, but not closed and difficult to solve in general.
Moreover:

- No information about cusps and other obstructions
- Wilson loops are functionals defined on the paths. But infinitesimal variation of a path doesn't necessarily yield infinitesimal variation of a functional

  @ [ICh, Mertens (2013) to appear]
- Variational analysis in the loop space is by no means straightforward

$$\mathcal{W}[\Gamma] = \mathcal{W}^{(0)} + \mathcal{W}^{(1)} = 1 - \frac{g^2 C_F}{2} \oint_\Gamma \oint_\Gamma dz_\mu dz'_\nu \ D^{\mu\nu}(z - z') + O(g^4)$$

$$D^{\mu\nu}(z - z') = -g^{\mu\nu} \ \Delta(z - z')$$

$$\Delta(z - z') = \frac{\Gamma(1-\epsilon)}{4\pi^2} \ \frac{(\pi\mu^2)^\epsilon}{[-(z-z')^2 + i0]^{1-\epsilon}}$$

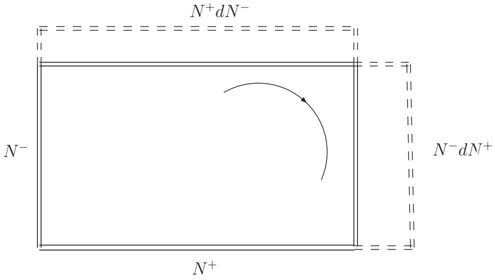
$$\frac{\delta \ \mathcal{W}[\Gamma]}{\delta\sigma_{\mu\nu}} == \frac{g^2 C_F}{2} \ \frac{\delta}{\delta\sigma_{\mu\nu}} \oint_\Gamma \oint_\Gamma dz_\lambda dz'^{\lambda} \ \Delta(z - z') + O(g^4)$$

Loop Space

Shape variations without the Stokes theorem

@ [ICh, Mertens, Van der Veken (2012,2013)]

Planar light-like rectangular Wilson loop: a “Hydrogen Atom” of the WL analysis



Loop Space

Shape variations without the Stokes theorem

@ [ICh, Mertens, Van der Veken (2012,2013)]

$$\mathcal{W}^{(1)}[\Gamma_\Box] = \frac{g^2 C_F}{2} \frac{\Gamma(1-\epsilon)(\pi\mu^2)^\epsilon}{4\pi^2} \cdot$$

$$\sum_{i,j} (v_i^\lambda v_j^\lambda) \cdot \int_0^1 \int_0^1 \frac{d\tau d\tau'}{[-(x_i - x_j - \tau_i v_i + \tau_j v_j)^2 + i0]^{1-\epsilon}}$$

$$2(v_1 v_2) = 2N^+ N^- \ , \ \frac{\delta}{\delta \ln \sigma} \equiv \sigma_{+-} \frac{d}{d\sigma_{+-}} + \sigma_{-+} \frac{d}{d\sigma_{-+}}$$

$$\frac{\delta \ \mathcal{W}[\Gamma_\Box]}{\delta \sigma_{\mu\nu}} = -\frac{\alpha_s N_c}{2\pi} \Gamma(1-\epsilon)(\pi\mu^2)^\epsilon \frac{\delta}{\delta \sigma_{\mu\nu}} (-2N^+ N^-)^\epsilon \ \frac{1}{2} \int_0^1 \int_0^1 \frac{d\tau d\tau'}{[(1-\tau)\tau']^{1-\epsilon}}$$

$$\mu \frac{d}{d\mu} \left[\frac{d}{d \ln \sigma} \ \ln \ \mathcal{W}[\Gamma_\Box] \right] = -\sum \Gamma_{\rm cusp}$$

area: $$\frac{d}{d\ln\sigma} \equiv \sigma^{\mu\nu}\frac{d}{d\sigma^{\mu\nu}} = \sigma^{+-}\frac{d}{d\sigma^{+-}} + \sigma^{-+}\frac{d}{d\sigma^{-+}}$$

$$d\sigma^{+-} = N^{+}dN^{-}\ ,\ d\sigma^{-+} = -N^{-}dN^{+}$$

rapidity: $$Y^{\pm} = \frac{1}{2}\ \ln\frac{(N^{\pm})^{+}}{(N^{\pm})^{-}} = \lim_{\eta^{\pm}\to 0} \pm\frac{1}{2}\ \ln\frac{N^{+}N^{-}}{\eta^{\pm}}$$

$$\frac{d}{d\ln\sigma} \sim \frac{d}{dY}$$

Conclusions II:

- Equations of motion in the Wilson Loop Space describe the reaction of the non-local functionals of the gauge fields to the shape variations of the paths in the underlying manifold
- In the subset of the cusped light-like paths, the geometrical evolution corresponds to the rapidity evolution
- Understanding evolution of the hadronic correlation functions via geometry of the WLS

Large-x_B Factorization

and evolution of transverse-distance dependent parton densities

@ [ICh, Mertens, Taels, Van der Veken (2013)]

- The struck quark acquires almost all momentum of the nucleon: $k_\mu \approx P_\mu$. Provided that the transverse component of the nucleon momentum is equal to zero, the transverse momentum of the quark $k_\perp$ is gained by the gluon interactions

  @ [Bassetto, Ciafaloni, Marchesini (1983); Korchemsky, Marchesini (1993)]

- A very fast moving quark with momentum k_μ can be considered as a classical particle with a (dimensionless) velocity parallel to the nucleon momentum P, so that the quark fields are replaced by the Mandelstam fields

$$\psi(0) = \mathcal{W}_P[\infty; 0] \, \Psi_{\rm in-jet}(0) \ , \ \bar\psi(z^-, z_\perp) = \bar\Psi_{\rm in-jet}(z) \, \mathcal{W}_P^\dagger[z; \infty]$$

$\bar\Psi_{\rm in-jet}, \Psi_{\rm in-jet}$ — incoming-collinear jets in the initial and final states

Large-x_B Factorization
and evolution of transverse-distance dependent parton densities

@ [ICh, Mertens, Taels, Van der Veken (2013)]

- Provided that almost all momentum of the nucleon is carried by the struck quark, real radiation can only be soft

$$q_\mu \sim (1 - x) P_\mu$$

- Virtual gluons can be soft or collinear, collinear gluons can only be virtual, quark radiation is suppressed in the leading-twist
- Rapidity singularities stem only from the soft contributions: they are known to occur at small gluon momentum $q^+ \to 0$. Rapidity divergences are known to originate from the minus-infinite rapidity region, where gluons travel along the direction of the outgoing jet, not incoming-collinear
- Real contributions are UV-finite (in contrast to the integrated PDFs), but can contain rapidity singularities and a non-trivial x_B- and $b_\perp$-dependence

Large-x_B Factorization
and evolution of transverse-distance dependent parton densities

Large-x_B factorization formula

$$\mathcal{F}\left(x, b_\perp; P^+, \mu^2\right) = \mathcal{H}(\mu, P^2) \times \Phi(x, b_\perp; P^+, \mu^2)$$

- $\mathcal{H}$ is x_B-independent, resums incoming-collinear partons
- Φ is the soft function

$$\Phi(x, b_\perp; P^+, \mu^2) = P^+ \int dx \; \mathrm{e}^{-i(1-x)P^+z^-} \cdot$$
$$\times \langle 0| \; \mathcal{W}_P^\dagger[z; -\infty]\mathcal{W}_{n^-}^\dagger[z; \infty]\mathcal{W}_{n^-}[\infty; 0]\mathcal{W}_P[0; \infty] \; |0\rangle$$

Rapidity and renormalzation-group evolution equations

$$\mu \frac{d}{d\mu} \ln \mathcal{F}\left(x, b_\perp; P^+, \mu^2\right) = \mu \frac{d}{d\mu} \ln \mathcal{H}(\mu^2) + \mu \frac{d}{d\mu} \ln \Phi(x, b_\perp; P^+, \mu^2)$$
$$P^+ \frac{d}{dP^+} \ln \mathcal{F}\left(x, b_\perp; P^+, \mu^2\right) = P^+ \frac{d}{dP^+} \ln \Phi(x, b_\perp; P^+, \mu^2)$$

The soft function $\mathcal{F}$ is a Fourier transform of an element of the (generalized) loop space. This fact enables us to consider the shape variations of this path, which are generated by the infinitesimal variations of the rapidity variable $\ln P^+$. The corresponding differential operator reads

$$\frac{d}{d \ln \sigma} \sim P^+ \frac{d}{dP^+} \ ,$$

Collins-Soper-Sterman rapidity-independent kernel

$$\mu \frac{d}{d\mu} \ \left(P^+ \frac{d}{dP^+} \ln \mathcal{F} \right) = \mu \frac{d}{d\mu} \ \left(P^+ \frac{d}{dP^+} \ln \Phi \right) =$$
$$= - \sum_{\mathrm{TDD}} \Gamma_{\mathrm{cusp}}(\alpha_s) = \mu \frac{d}{d\mu} \mathcal{K}_{\mathrm{CSS}}(\alpha_s)$$

Outlook:

- Theoretical and phenomenological study of the **three-dimensional structure of nucleons** with the **Wilson lines/loops formalism** as the main instrument
- Mathematical structure of the **loop space**: gauge-invariant formulation of the TMDs in terms of the nucleon matrix elements; complete evolution of the TMDs from geometrical properties of the loop space
- Ultimate goal: field-theoretically motivated **dynamical 3D-picture of the nucleon**; the fundamental problem of the nucleons spin composition from the quark and gluon constituents

Preliminary Results:

1. I.O. Cherednikov, T. Mertens, P. Taels, F.F. Van der Veken,
"Evolution of transverse-distance dependent parton densities at large-x_B and geometry of the loop space"
Int. J. Mod. Phys. Conf. Ser. (2013) [in print]; `arXiv:1308.3116 [hep-ph]`

2. I.O. Cherednikov, J. Lauwers, P. Taels,
"On a Wilson lines approach to the study of jet quenching"
`arXiv:1307.5518 [hep-ph]`

3. I.O. Cherednikov,
"On singularities of the TMDs, their origin and treatment"
Nuovo Cim. C36 (2013) 215

4. I.O. Cherednikov, T. Mertens, F.F. Van der Veken,
"Cusped light-like Wilson loops in gauge theories"
Phys. Part. Nucl. 44 (2013) 250; `arXiv:1210.1767 [hep-ph]`

5. F.F. Van der Veken, I.O. Cherednikov, T. Mertens,
"Evolution and dynamics of cusped light-like Wilson loops in loop space"
AIP Conf. Proc. 1523 (2013) 272; `arXiv:1212.4345 [hep-th]`

6. I.O. Cherednikov, T. Mertens, F.F. Van der Veken,
"Loop space and evolution of the light-like Wilson polygons"
Int. J. Mod. Phys. Conf. Ser. 20 (2012) 109; `arXiv:1208.5410 [hep-th]`

7. I.O. Cherednikov, T. Mertens, F.F. Van der Veken,
"Evolution of cusped light-like Wilson loops and geometry of the loop space"
Phys. Rev. D86 (2012) 085035; `arXiv:1208.1631 [hep-th]`