Lightlike string-localized fields: the example of massive scalar QED

José Gracia-Bondía
(jointly with J. Mund and J. Várilly)

Nijmegen, 4 April 2016

A funny type of quantum field

A string-localized or SLF field $A(x,l)$ for particles of any spin j (including infinite) and mass ≥ 0 is an operator-distribution on Fock-Hilbert space, depending on coordinates and on half-strings reaching the spatial or null *infinity*, with properties:

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- A given link to "ordinary" quantum fields.

A stringy *vector* field I

SLF = "stringy" fields become interesting from $j = 1$ onwards.

Let us take up this case. A dreibein $e_r(p)$ on Minkowski momentum space, with the properties:

$$\big(e_r(p)\, e_s(p)\big) = -\delta_{rs} \quad \text{for} \quad r, s = 1, 2, 3; \qquad \big(p\, e_r(p)\big) = 0,$$

describes polarization states for particles with mass $m^2 = p^2 > 0$ and spin $j = 1$.

Using e_r, one can construct a free skewsymmetric tensor field acting on their Fock space, by the formula:

$$F^{\mu\nu}(x) := i \sum_r \int d\mu(p) \Big[e^{i(px)} \big(p^\mu e_r^\nu(p) - p^\nu e_r^\mu(p) \big) a_r^\dagger(p) \\ - e^{-i(px)} \big(p^\mu e_r^\nu(p)^* - p^\nu e_r^\mu(p)^* \big) a_r(p) \Big]. \qquad (1)$$

A stringy *vector* field II

A stringy vector field, or "vector potential" if you wish, is then defined on the same Fock space as:

$$A^{\mu}(x,l) := \int_0^{\infty} dt\, F^{\mu\nu}(x+tl)\, l_{\nu}\,. \tag{2}$$

There are variants, but we concentrate on Eq. (2). With null strings, the definition depends only on the ray of l –or the light front in the Dirac sense uniquely associated to it.

This field has

- The linking property:[1]

$$dA(x,l) = F(x).$$

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- The main thing: covariance, which bears repetition:

$$U(a,\Lambda)A^\mu(x,l)U^\dagger(a,\Lambda) = A^\nu(\Lambda x + a, \Lambda l)\,\Lambda_\nu{}^\mu. \qquad (3)$$

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It would not be exaggerated to claim that perturbative QFT since 1970 has turned around the renormalization of "gauge" theories, with the attendant congeries of Faddeev–Popov ghosts, mutating into the global supersymmetry discovered by Becchi, Rouet and Stora, which in turn became a happy hunting ground for mathematical physicists...

Getting explicit

The formula for (massive) $A(x,l)$ is as follows:

$$A^\mu(x,l) = \sum_r \int d\mu(p)\Big[e^{i(px)}\, u_r^\mu(p,l)\, a_r^\dagger(p) + e^{-i(px)}\, u_r^\mu(p,l)\, a_r(p)\Big],$$

where:

$$u_r^\mu(p,l) := e_r^\mu(p) - p^\mu \frac{(e_r(p)\,l)}{(pl)},$$

with $e_r^\mu(p)$ being the polarization dreibein we started from, and the intertwining property holds:

$$D^{(j=1)}(R(\Lambda,p))\, u(\Lambda^{-1}p, \Lambda^{-1}l) = u(p,l)\; D^{(j_1=\frac{1}{2}, j_2=\frac{1}{2})}(\Lambda) = u(p,l)\Lambda,$$

where $R(\Lambda,p)$ is the "Wigner rotation".

The Wightman connection

What (else) we do win by this? Look at the two-point functions. For the pointlike analogue of the electromagnetic potential (Proca field) $B^\mu(x)$ they are of the form:

$$\langle 0 \,|\, B_\mu(x)\, B_\nu(x') \,|\, 0 \rangle \propto \int d\mu(p)\, e^{-ip(x-x'))} \Big(-g_{\mu\nu} + p_\mu p_\nu/m^2\Big);$$

threatening us with a propagator of the form $\frac{-g_{\mu\nu}+p_\mu p_\nu/m^2}{p^2-m^2}$; and indeed the Proca field has lousy ultraviolet properties, needing the dimension-lowering properties of indefinite metrics…

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leading to a propagator which scales exactly as the one of scalar particles. This feat you can perform for any spin.

Philandering fields

The theory of stringy fields has been developed as a branch of algebraic quantum field theory,[2] by several authors: Borchers, Guido, Longo, Rehren, Schroer, Yngvason among them.

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However, here we are interested in interacting models. We seek a *principle* and a *method* that allows the perturbative construction of such models. I show my cards:

- The *principle* is stated simply enough: **string-freedom** of the "physical" amplitudes.
- The only *method* I see as up to the task is (a version of) the Epstein-Glaser (EG) renormalization scheme.

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Choosing models

In view of $dA = F$, it is not hard to see that there must be a (non-unique) scalar "escort" field $\phi(x,l)$ living on the same Fock space as F and A,[3] such that

$$d_l A = d_l \partial_\mu \phi = \partial_\mu d_l \phi =: \partial_\mu w, \quad \text{where} \quad d_l = \sum dl_\mu \frac{\partial}{\partial l_\mu}.$$

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A way to have string-freedom is to **couple** $A^\mu(x,l)$ with a conserved pointlike current $j_\mu(x)$ so $d_l(Aj) = (\partial wj) = \partial(wj) =: Q$.

This divergence will not contribute to the physical quantities.

As an example, we consider here massive scalar QED, in which a conserved charged current $j_\mu(x) = {:}\varphi^\dagger(x) \overleftrightarrow{\partial_\mu} \varphi(x){:}$ couples with a massive "photon" –I steer clear of infrared troubles for now.[4]

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[4] Although stringy fields are promising in this respect.

But why would one do that?

It is neither trivial nor obvious that string-freedom survives renormalization: here the field theorist *must earn her bread!* Before embarking on the renormalization trip, it is good to summarize the advantages of stringy fields.

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  2. Not unrelated to the above: better ultraviolet behaviour, taking place irrespectively of spin.
  3. More generality: Wigner's infinite-helicity particle with Casimir values $P^2 = 0$ and $W^2 < 0$ enters the realm of physics. This particle would be inert, except under the action of gravity.

The young Ingvason

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Zero-Mass Infinite Spin Representations of the Poincaré Group and Quantum Field Theory

JAKOB YNGVASON

Institut für Theoretische Physik, Universität Göttingen

Received December 15, 1969

Abstract. It is shown that a local quantized field with a manifestly covariant transformation law under the Poincaré group cannot have nonvanishing matrix elements between the vacuum and an irreducible subspace of zero mass and infinite spin.

Jakob Yngvason in 1970 published a no-go theorem on the existence of a local quantum field for the Wigner infinite-helicity unirrep on a Poincaré module.

The old Yngvason, with coauthors

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String-localized quantum fields from Wigner representations

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Abstract

In contrast to the usual representations of the Poincaré group of finite spin or helicity the Wigner representations of mass zero and infinite spin are known to be incompatible with point-like localized quantum fields. We present here a construction of quantum fields associated with these representations that are localized in semi-infinite, space-like strings. The construction is based on concepts outside the realm of Lagrangian quantization with the potential for further applications.

PACS: 03.70.+k; 11.10.Cd; 11.10.Lm; 11.30.Cp

The same Yngvason in 2004, together with Mund and Schroer, showed that there is a (spacelike) SL quantum field for this Wigner stuff. (A similar proof works for lightlike ones.)

As a matter of taste...

- **Subjective advantages** (and wild hopes).
  4. We get rid of the curses of the gauge (and "gauge-symmetry breaking" nightmares). Quantum fields are *restored* as mediators between the causal localization principles of QFT and the measurable world of particles.

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6. A smoother transition to massless models —we see further down a nonassuming example.
7. Could the mentioned Wigner stuff be a component of **dark matter**?

On the generality of the construction

There is nothing special about $s = 1$.

Recall that $F_{\mu\nu}(x)$ corresponds to the direct sum $(1,0) \oplus (0,1)$ of $SL(2,\mathbb{C})$ irreps. Consider (for the moment massive) particles of spin $j = 2$. There is a field with the Lorentz transformation type $(2,0) \oplus (0,2)$, a **fourth-rank tensor** $R_{\mu\nu\sigma\lambda}(x)$ similar to Riemann–Christoffel curvature: skewsymmetric within each pair of indices and symmetric bewtween the pairs. This results of of applying a differential operator (now of second order) to a **symmetric, traceless, divergence-free 2-tensor, say** $h_{\mu\nu}(x)$ —of $(1,1)$. Surprisingly, this "potential" and the "field" appear to enjoy the same scaling properties, better than the ones of the other potential, but not good enough for renormalizability. Then one constructs from $R_{\mu\nu\sigma\lambda}$ a SL field which scales like a scalar particle.[5] It all **works the same** for any integer spin.

[5] See forthcoming work by Mund and de Oliveira.

Towards calculation

Reminder: in the EG approach one postulates a perturbative expansion of the $\mathbf{S}[g]$-matrix as an OVD, of the form:

$$\mathbf{S}[g] = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{\mathbb{M}} d^4x_1 \cdots \int_{\mathbb{M}} d^4x_n \, T_n(x_1, \dots, x_n; l) \, g(x_1) \cdots g(x_n).$$

The T_n (unbounded OVD) are called *time-ordered n-point functions*. It is expected that in the adiabatic limit $g \to 1$ the $\mathbf{S}[g]$ matrix will tend to the physical $\mathbf{S}$-matrix.

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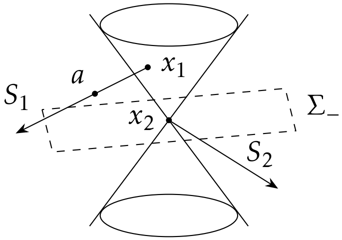
One tries to determine the T_n by some natural prescriptions:

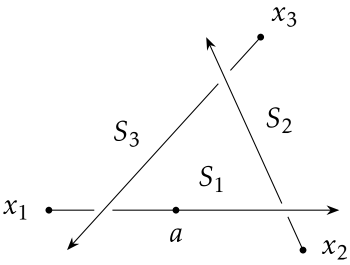
1. symmetry in their spacetime coordinates;
2. covariance;
3. causality;
4. Wick expansion rule;
5. last but not least, *perturbative string independence*.

A warning

To perform recursive causal expansions, as required in the EG method, with **spacelike** strings is tricky. You can organize second-order computations…



Causally separating two half-strings…



…but with three?

The problem vanishes with null strings.

Go forth and compute

We exemplify within our chosen model, with vertex: $:\varphi^\dagger(x)\overleftrightarrow{\partial}_\mu\varphi(x){:}A(x,l)$. Scalar QED is much more instructive than spinor QED.

Let me exhibit some second-order calculations. Since Wick's theorem applies, one has for $\mathbf{S}^{(2)}(l)$:

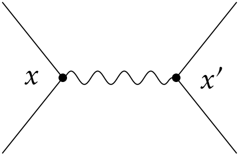
$$-\frac{e^2}{2}\iint_{\mathbb{M}^2} d^4x\,d^4x'\,T_2(x,x';l) \sim -\frac{e^2}{2}\iint_{\mathbb{M}^2} d^4x\,d^4x'\,\mathrm{T}[\mathcal{L}(x,l)\mathcal{L}(x',l)]$$

$$=: -\frac{e^2}{2}\iint_{\mathbb{M}^2} d^4x\,d^4x'\Big[\mathbf{S}^{(2)}_{(0,0)}+\mathbf{S}^{(2)}_{(1,0)}+\mathbf{S}^{(2)}_{(0,1)}+\mathbf{S}^{(2)}_{(1,1)}+\mathbf{S}^{(2)}_{(0,2)}+\mathbf{S}^{(2)}_{(1,2)}\Big],$$

Trvially, for the disconnected part:

$$\begin{aligned} d_l\mathbf{S}^{(2)}_{(0,0)} &= d_l{:}A^\mu(x,l)A^\nu(x',l){:}\,{:}j_\mu(x)j_\nu(x'){:} \\ &= \partial^\mu\Big(Q_\mu(x,l)\,\mathcal{L}(x',l)\Big)+\partial'^\mu\Big(\mathcal{L}(x,l)\,Q_\mu(x',l)\Big). \end{aligned}$$

Tree graphs I

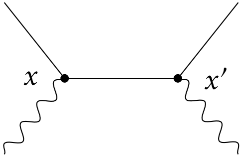


$\mathbf{S}^{(2)}_{(1,0)} \sim$ "Møller" scattering

$$:j_\mu(x) j_\nu(x'): d_l \langle 0 \,|\, \mathrm{T}_0 (A^\mu(x,l) A^\nu(x',l)) \,|\, 0 \rangle = :j_\mu(x) j_\nu(x'):$$
$$\times \Big(\partial^\mu \langle 0 \,|\, \mathrm{T}_0 \, w(x,l) A^\nu(x',l) \,|\, 0 \rangle + \partial'^\nu \langle 0 \,|\, \mathrm{T}_0 \, A^\mu(x,l) w(x',l) \,|\, 0 \rangle \Big).$$

The second equality is not trivial, but is not difficult. Nothing untoward happens with $\langle 0 \,|\, \mathrm{T}_0 (A^\mu(x,l) A^\nu(x',l)) \,|\, 0 \rangle$, and here this is precisely the point.

Tree graphs II



$\mathbf{S}^{(2)}_{(0,1)} \sim$ "Compton" scattering

Matters are even more interesting here. The line $x \leftrightarrow x'$ contains second order derivatives of the Feynman propagator. This yields a one-parameter ambiguity in the renormalization.

$$\langle 0 \,|\, \mathrm{T}\, \partial_\mu \varphi(x)\, \partial_\nu \varphi^\dagger(x') \,|\, 0 \rangle = \langle 0 \,|\, \mathrm{T}\, \partial_\mu \varphi^\dagger(x)\, \partial_\nu \varphi(x') \,|\, 0 \rangle$$
$$:= \partial_{\mu\nu} D_F(x - x') + C g_{\mu\nu}\, \delta(x - x').$$

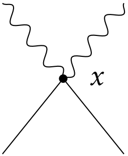
As it turns out, string-independence forces the replacement $C = -1$.

By the seashore

With that coefficient of $\delta(x-x')$, there appears a new, **local** term in $\mathbf{S}^{(2)}_{(0,1)}$, to wit:

$$2(A^\mu(x,l)A^\nu(x',l))\,\varphi^\dagger(x)\varphi(x')\,\delta(x-x'),$$

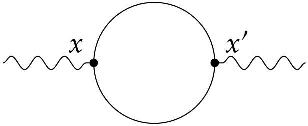
clearly yielding the "seagull" in massive scalar QED (exactly the same as for true photons).



The seagull

In other words: the seagull graph is required by internal consistency of our formalism.

Vacuum polarization



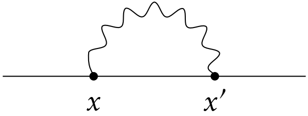
$\mathbf{S}^{(2)}_{(0,2)} \sim$ vacuum polarization

Vacuum polarization: this quadratic divergence is essentially trivial for our purpose. The stringy field in the external legs leaves no freedom.
We only need to ascertain, in the renormalization process, that

$$\partial_\nu \langle 0 \,|\, \mathrm{T}\, j^\mu j'^\nu \,|\, 0\rangle = 0 = \partial_\mu \langle 0 \,|\, \mathrm{T}\, j^\mu j'^\nu \,|\, 0\rangle.$$

By (exclusive) use of the so-called "central solution" to the extension problem of distributions in the EG procedure, defined by normalization at $p = 0$, this holds.

Self-energy of the selectron



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This graph is also quadratically divergent here. There are many not point-local terms to compute, and I have not finished the job. At any rate, the strongly supported conjecture is: the central solution does respect string-independence. But it is **not unique** in that.

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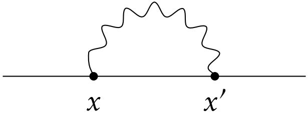

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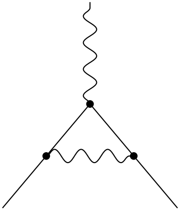
As a general rule of thumb: the *external "photon" legs* a graph *govern* the set of admissible renormalized solutions. The more such legs a graph has, the more constrained is it by string-independence.

Moving along…

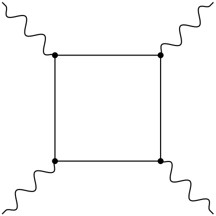


First correction to the 3-point function

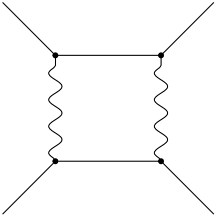
Next in line here is the vertex correction, with an attendant **Ward–Takahashi-like identity** relating it to self-energy, from string-independence.

(Eventually, a recursive EG-style proof of renormalizability at all orders.)

Look at these 4-point graphs!



Photon-photon scattering



A selectron-selectron vertex

The primitive logarithmic divergence on the left is **very constrained**, a unique solution to the string-freedom requirement is strongly conjectured. Thus we can speak of a radiative process of photon-photon scattering.

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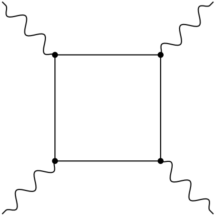
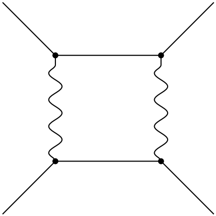


Photon-photon scattering



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On the other hand, I regard the graph on the right as defining a new local vertex.

Model incomplete

Graphs with *four external selectron legs* are logarithmically divergent and generate an undetermined (re)normalization constant, that is, a new **local** vertex. In other words: the (complex) φ^4 theory is automatically contained in scalar QED; scalar electrodynamics is not a complete theory.

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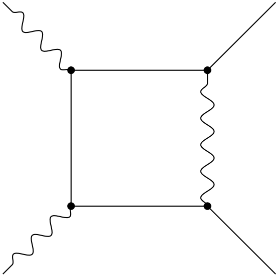
In the standard treatment, such graphs are logarithmically divergent **only** for massless photons, whereas for *massive* photons they are quadratically divergent by power counting.

Part of the alchemy of the SL formalism is that here there is no difference between the massless and the massive case, in the present respect: we only have to deal with logarithmic divergences.

The other 4-point graph



Not to be missed. . .

The divergences of the Compton or spositron-selectron annihilation type graphs are **strongly** constrained, too; so here we speak of radiative corrections, as well.

What the future may bring

I might have succeeded in calling your attention on the fun, interest and feasibility of working with SL fields.

But of course the real prize is in dealing with (the customarily called) non-Abelian Yang-Mills fields, massive and massless, of the Standard Model. The Lie-algebraic structure is a result of positivity, as many other features, via string freedom.

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And that's all, folks!

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