**C. Pinzari**, *Quantum Groups and Conformal Field Theory*
The notion of weak quasi-Hopf algebra was introduced by Mack and Schomerus as an extension of that of quasi-Hopf algebra due to Drinfeld to study WZW models of Conformal Field Theory. In my talk we shall discuss how weak quasi-Hopf algebras may turn useful to connect these models with quantum groups at roots of unity and we shall discuss questions of unitarizability or equivalences of their representation categories. The talk is based on a joint work with Sebastiano Carpi and Sergio Ciamprone.

**J.-L. Sauvageot**, *Listening to the shape of a drum*
We show that the Möbius group $G(\mathbb{R}^n)$ of a Euclidean space $\mathbb{R}^n$ having dimension $n \geq 3$ acts isometrically on the multipliers algebra $\mathcal{M}(H_e^{1,2}(\mathbb{R}^n))$ of the extended Sobolev space $H_e^{1,2}(\mathbb{R}^n)$. Then we prove that the Dirichlet integral $(\mathcal{D}, H_e^{1,2}(\mathbb{R}^n))$ is a closable quadratic form on the space $L^2(\mathbb{R}^n, \Gamma[a])$ associated to the energy measure $\Gamma[a] = |\nabla a|^2 dx$ of any multiplier $a$. Moreover, it is shown that its quadratic form closure $(\mathcal{D}, \mathcal{F}^a)$ is a Dirichlet form on $L^2(\mathbb{R}^n, \Gamma[a])$ which is naturally unitarily equivalent to the one $(\mathcal{D}, \mathcal{F}^{a\circ\gamma})$ on $L^2(\mathbb{R}^n, \Gamma[a \circ \gamma])$ for any Möbius transformation $\gamma$. In a conferse direction we prove that an homeomorphism $\gamma : U \to \gamma(U)$ which gives rise to an algebraic isomorphism $a \mapsto a \circ \gamma$ between the algebras of finite energy multipliers $\mathcal{FM}(H^{1,2}(\gamma(A)))$ and $\mathcal{FM}(H^{1,2}(A))$ of any relatively compact domain $A \subseteq U$ and leaves invariant the corresponding fundamental tones, $\mu_1(\gamma(A), a) = \mu_1(A, a \circ \gamma)$, necessarily is the restriction to $U$ of a Möbius transformation. Companion conclusions are proved for quasi-conformal and bounded distortion maps. These results are preceded by the study of the connections between fundamental tones and ergodic properties of multipliers. In particular, it is shown that the non vanishing of the fundamental tone of any multiplier follows from the existence of the spectral gap of the Dirichlet integral.

**G. Skandalis**, *Stability of Lie groupoid C*-algebras and of C*-algebras of singular foliations*
We will prove that if the anchor map is nowhere zero, the C*-algebra of a Lie groupoid is always stable We will also consider the analogous result for singular foliations. This is based on a joint paper with Claire Debord

**R.J. Szabo**, *Noncommutative gauge theories and non-geometric backgrounds*
We describe the noncommutative gauge theories arising on the worldvolumes of D-branes in non-geometric string backgrounds, focusing in detail on the cases of three-dimensional T-folds. The T-duality monodromies of the non-geometric backgrounds lead to Morita duality monodromies of the noncommutative Yang-Mills theories induced on the D-branes. For some classes of T-folds we recover the well-known examples of noncommutative principal torus bundles from topological T-duality, while others give new examples of noncommutative fibrations with non-geometric torus fibres.

**J. Várilly**, *Canonical and covariant treatments of Wigner particles*
In recent years there has been renewed interest in the so-called "continuous spin" representations of the Poincaré group, identified by Wigner in 1939 but as yet unobserved. Canonical approaches, via the coadjoint orbit picture or the little-group method, yield rough descriptions of these particles; and Wigner later wrote down wave equations to describe them. Here we outline how to go from the little-group picture to the covariant wave equations, as a first step towards their field theory.

## Schedule

| | **Mon 16** | **Tue 17** | **Wed 18** | **Thu 19** | **Fri 20** |
|---|---|---|---|---|---|
| 8:45 - 9:00 | Opening | | | | |
| 9:00 - 10:00 | Longo | Várilly | Marmo | Skandalis | Khalkhali |
| 10:00 - 10:30 | Coffee | Coffee | Coffee | Coffee | Coffee |
| 10:30 - 11:30 | Bruzzo | Dabrowski | Lizzi | Majid | Arnlind |
| 11:30 - 12:30 | Nest | Brzezinski | Dubois-Violette | Bartocci | Sauvageot |
| 13:00 - | Lunch | Lunch | Lunch | Lunch | Lunch |
| 17:00 - 18:00 | Kaad | 16:30 - 17:30 Bieliavsky | Aschieri | Paycha | |
| 18:00 - 19:00 | Pinzari | 17:30 - 18:30 Ciccoli | Szabo | Barrett | |
| | | 18:30 - 19:30 De Nittis | Bahns | | |
| 19:30 - | Dinner | Dinner | Dinner | Dinner | |

# Noncommutative manifolds and their symmetries

A conference dedicated to Giovanni Landi on the occasion of his 60th Birthday


Francesca Arici
Alain Connes
Francesco D'Andrea
Giuseppe Dito
Chiara Pagani
Walter van Suijlekom


Scalea, 16–20 September 2019

## Abstracts

**J. Arnlind**, *Pseudo-Riemannian calculi and Noncommutative Minimal Submanifolds*
Pseudo-Riemannian calculus was developed as a framework for studying noncommutative metric and torsion-free connections and, in this setting, one may show that a unique Levi-Civita connection exists. Several standard noncommutative manifolds can be described via pseudo-Riemannian calculi and there exists a theory of curvature much in analogy with the classical theory. In this talk, I will give a brief overview of pseudo-Riemannian calculi and show that one may correspondingly define submanifolds in the spirit of differential geometry, including analogues of the second fundamental form, the Weingarten map and Gauss' equations. In particular, we propose a definition of mean curvature and minimal submanifolds, and show that the noncommutative torus is minimally embedded in the noncommutative 3-sphere.

**P. Aschieri**, *Cartan's structure equations and Levi-Civita connection in braided geometry*

**D. Bahns**, *The G-wavefront set and the twisted convolution product*
We give a sufficient criterion for the existence of the twisted convolution product of two tempered distributions as a tempered distribution, and we list examples of algebras with respect to this and related products contained in $S'$. (joint work with René Schulz)

**J. Barrett**, *Fuzzy spaces as noncommutative manifolds*
The talk will report progress on understanding spectral triples for fuzzy spaces as non-commutative analogues of manifolds. It will explain

some particular examples, such as the fuzzy torus, and then look at recent results on random finite spectral triples.

**C. Bartocci**, *Poisson-Nijenhuis structures on quiver algebras*
After introducing some basic concepts, the definition of dynamical system on the path algebra of a quiver will be given. In particular, the notion of bi-hamiltonian system will be generalized to this context. Some significant examples will be discussed.

**P. Bieliavsky**, *Geometric unitary dual 2-cocycles for a class of locally compact groups*
I will describe some geometric constructions that produce locally compact quantum groups.

**U. Bruzzo**, *An exercise on the McKay correspondence in 3 dimensions*
If $G$ is a finite group acting on $C^n$, the MacKay correspondence establishes a correspondence between the representation theory of $G$ and the cohomology of a crepant resolution $X$ of $C^n/G$, or more precisely, with the geometry of the exceptional divisors of $X$. In my talk I will cover the following aspects: 1) correspondence between the GIT construction of the resolution vs. a Marsden-Weinstein approach; 2) explicit study of the chamber structure of the space of stability parameters in an example; 3) a hint to physical applications.

**T. Brzezinski**, *Towards noncommutative bundles with homogeneous fibres*
We present an algebraic framework for constructing and studying noncommutative bundles with quantum homogeneous spaces, such as Podles' 2-spheres, as fibres. Specific examples include the two-parameter deformation of the flag manifold interpreted as a bundle over the quantum projective plane with the general Podles' 2-sphere as a fibre, and the quantum twistor space viewed as a bundle over the quantum 4-sphere with the standard Podles' sphere as a fibre. This is a joint work with Wojciech Szymanski (Odense).

**N. Ciccoli**, *Quantum orbit method*

**L. Dabrowski**, *Noncommutative inner geometry of the Standard Model*
It is customary to regard a non-commutative C*-algebra as the algebra of continuous functions on a 'quantum space'. Its smooth and metric structures can be described in terms of a spectral triple which involves an analogue of the Dirac operator. I will interpret the inner part of the almost commutative geometry of the Standard Model of fundamental particles in physics as a quantum analogue of the de-Rham-Hodge spectral triple.

**G. De Nittis**, *Spectral Continuity for Aperiodic Quantum Systems*
How does the spectrum of a Schrödinger operator vary if the corresponding geometry and dynamics change? Is it possible to define approximations of the spectrum of such operators by defining approximations of the underlying structures? In this talk a positive answer is provided using the rather general setting of groupoid C*-algebras. A characterization of the convergence of the spectra by the convergence of the underlying structures is proved. In order to do so, the concept of continuous field of groupoids is used. The approximation scheme is expressed through the tautological groupoid, which provides a sort of universal model for fields of groupoids. The use of the Hausdorff topology turns out to be fundamental in understanding why and how these approximations work. The construction presented during the talk is adapted to the case of Schrödinger operator with Delone potential (i.e. quasi-crystals). The talk is based on a joint work with: S. Beckus and J. Bellissard

**M. Dubois-Violette**, *Finite quantum geometry and fundamental particles*
We show that the spectrum of fundamental particles of matter and their symmetries can be encoded in a finite quantum geometry equipped with a supplementary structure connected with the quark-lepton symmetry. The occurrence of the exceptional quantum geometry for the description of the standard model with 3 generations is suggested. We discuss the field theoretical aspect of this approach taking into account the theory of connections on the corresponding Jordan modules.

**J. Kaad**, *On the unbounded picture of KK-theory*
One of the powers of KK-theory lies in the mixture of good abstract properties and the abundance of explicit classes. The prototypical examples of classes in KK-theory come from differential geometry being Dirac operators (for K-homology) or symbols of Dirac operators (for K-theory). A common feature for these examples is that they are in fact given by unbounded operators and hence belong to the unbounded picture of KK-theory. The link between the unbounded picture of KK-theory and the Kasparov picture of KK-theory is established by the Baaj-Julg bounded transform, and this transform turns out to be surjective under a mild separability condition. In this talk I'll explain how to describe the kernel of the Baaj-Julg bounded transform via an equivalence relation that can be introduced at the unbounded level using homotopies of unbounded operators and a novel notion of spectral decomposability.

**M. Khalkhali**, *From random spectral triples to spectral curves and topological recursion*
Recently suggested matrix models to probe quantum gravity based on Dirac operators on finite spectral triples, pose very challenging analytic problems. In particular their large N limits have only been studied by computer simulations. There are also conjectures about existence of phase transition in the limit laws. In this talk I will show how new techniques developed in modern random matrix theory, namely topological recursion and the theory of Riemann surfaces (more precisely spectral curves), can be effectively applied and yield rigorous results (even for more general models). The Schwinger-Dyson equations satisfied by the connected correlators $W_n$ of the corresponding multi-trace formal 1-Hermitian matrix model are derived. I will show that the coefficients $W_{g,n}$ of the large N expansion of $W_n$'s enumerate discrete surfaces, called stuffed maps, whose building blocks are of particular topologies. The spectral curve $(\Sigma, \omega_{0,1}, \omega_{0,2})$ of the model can be investigated in detail. In particular, I will give an explicit expression for the fundamental symmetric bidifferential $\omega_{0,2}$ in terms of the formal parameters of the model. This is a joint work with S. Azarfar. (arXiv:1906.09362).

**F. Lizzi**, *Journey of a Fellow Traveller*
I will give very personal physicist view of noncommuntative geometry, as I learned and practised with Gianni across two millennia.

**R. Longo**, *The information in a wave*

**S. Majid**, *Quantum gravity on a quadrilateral and other topics*
Using noncommutative geometry, we quantise gravity on a quadrilateral. We also consider 1+0 quantum scalar field theory on a lattice line with arbitrary metric on the edges and plausibly compute cosmological particle creation for a jump metric. If time I may also mention recent work on Poisson principal bundles.

**G. Marmo**, *Geometry, Quantum Mechanics, and Information Theory*
By using the geometrical formulation of quantum mechanics, we show how to induce geometrical structures on submanifolds of quantum states. As the Fisher-Rao metric turns out to be a restriction of the quantum Fubini-Study metric to an isotropic and totally geodesic submanifold, we argue that quantum mechanics provides a generalization of information geometry.

**R. Nest**, *Around the functional equation*
The functional equation for the Riemann zeta function is based on analysis of asymptotic behaviour for $t \approx 0$ of expression like $Tr(\exp(-tD^2))$, where $D$ is, say, an elliptic operator on a smooth closed manifold $M$. In particular, it depends heavily on the fact that the expressions like $Tr(\exp(-tD^2))$ have Melin transform which is holomorphic on a subspace of the complex plane of the form $Re(z) > C$, which is a consequence of finite dimensionality of $M$. We will construct an analogue of the meromorphic extension of the Riemann zeta function and prove the corresponding functional equation in the infinite dimensional limit case. We will sketch some work in progress which give applications of these constructions to local index formulas for operators associated to infinite dimensional physical systems.

**S. Paycha**, *Meromorphic germs with linear poles and the geometry of cones*
Meromorphic germs in several variables whose poles are linear at zero arise in various situations involving renormalisation. Feynman integrals, multizeta functions and their generalisations, namely discrete sums on cones and discrete sums associated with trees give rise to meromorphic germs at zero with linear poles. One typically aims at evaluating them at zero to renormalise the a priori divergent multiple integrals and sums. In order to preserve a locality principle reminiscent of quantum field theory, the evaluation at zero should factor on functions with independent sets of variables. Inspired by Speer, we shall discuss a class of generalised evaluators which do the job. Using a theory of Laurent expansions on meromorphic germs with linear poles at zero, we shall relate these generalised evaluators to the geometry of cones. This talk is based on joint work with Pierre Clavier, Li Guo and Bin Zhang.